11. Gears

Objectives
- Understand basic principles of gearing.
- Understand gear trains and how to calculate ratios.
- Recognize different gearing systems and relative advantages and disadvantages between them.
- Understand geometry of different gears and their dimensional properties.
- Recognize different principles of gearing.
- Recognize the unorthodox ways gears can be used in different motion systems.

Introduction
- Gears are the most common means used for power transmission.
- They can be applied between two shafts which are:
  - Parallel
  - Collinear
  - Perpendicular and intersecting
  - Perpendicular and nonintersecting
  - Inclined at any arbitrary angle
- Gears are made to high precision. Purchased from gear manufacturers rather than made in house.
- However, it is necessary to design for a specific application so that proper selection can be made.
- Used to be called toothed wheels dating back to 2600 b.c.

An 18th Century Application of Gears for Powering Textile Machinery

11.2 Types of Gears

- **Spur gears**
  - Internal gears
  - Most common form
  - Used for parallel shafts
  - Suitable for low to medium speed application
  - Relatively high ratios can be achieved (< 7)
  - Steel, brass, bronze, cast iron, and plastics
  - Can also be made from sheet metal

**Gear Parameters**

- Number of teeth
- Form of teeth
- Size of teeth
- Face Width of teeth
- Style and dimensions of gear blank
- Design of the hub of the gear
- Degree of precision required
- Means of attaching the gear to the shaft
- Means of locating the gear axially on the shaft

**Gear Types**

- Helical gears
  - Teeth are at an angle
  - Used for parallel shafts
  - Teeth engage gradually reducing shocks
Helical Gears

Helical Gear

Helical Gear Characteristics
- Helix angle 7 to 23 degrees
- More power
- Larger speeds
- More smooth and quiet operation
- Used in automobiles
- Helix angle must be the same for both the mating gears
- Produces axial thrust which is a disadvantage

Herringbone Gears
- Two helical gears with opposing helical angles side-by-side
- Axial thrust gets cancelled

Herringbone Gears

Herringbone Gear
Herringbone Gear Machining

Gear Types
- Bevel gears
  - They have conical shape

Gear Types
- Worm gears
  - For large speed reductions between two perpendicular and non-intersecting shafts
  - Driver called worm looks like a thread

Bevel Gears (Miter gears)
- For one-to-one ratio
- Used to change the direction

Bevel Gears
Rack and pinion

- A rack is a gear whose pitch diameter is infinite, resulting in a straight line pitch circle.
- Involute of a very large base circle approaches a straight line
- Used to convert rotary motion to straight line motion
- Used in machine tools

Internal spur gear

- Provides more compact drives compared to external gears
- They provide large contact ratio
- Relatively less sliding and hence less wear compare to external gears
Internal spur gear

Internally Meshing Spur Gears

Figure 14.14 Internally meshing spur gears.


Gear Assemblies

- Identified based on the input and output shaft positions

<table>
<thead>
<tr>
<th>Parallel shaft</th>
<th>Perpendicular shaft</th>
<th>Other types</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spur gears</td>
<td>Bevel and Miter gears</td>
<td>Rack-and-pinion</td>
</tr>
<tr>
<td>Helical gears</td>
<td>Cross-helix</td>
<td></td>
</tr>
<tr>
<td>Worm gears</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Other types

- Perpendicular shaft

- Parallel shaft

Velocity Ratio

- Velocity ratio is defined as the ratio of rotational speed of the input gear to that of the output gear

\[ V_r = \frac{N_g}{N_p} = \frac{D_g}{D_p} \]

Velocity Ratio

- \( V_r \) = Velocity ratio
- \( N_p \) = Number of teeth on pinion
- \( N_g \) = Number of teeth on gear
- \( D_p \) = Pitch diameter of pinion
- \( D_g \) = Pitch diameter of gear

Fig. 11-9 Velocity Ratio

\[ V_r = \frac{N_g}{N_p} = \frac{D_g}{D_p} \]
Example Problem 11-1: Velocity Ratios and Gear Trains

• For the set of four gears shown below, calculate output speed, output torque, and horsepower for both input and output conditions and overall velocity ratio:

Example Problem 11-1: Velocity Ratios and Gear Trains
(cont'd.)

Example Problem 11-2: Velocity Ratios and Gear Trains

• For the gear train shown below, determine the train value, output speed, output direction, output torque, and output power:

Example Problem 11-2: Velocity Ratios and Gear Trains
(cont'd.)
Spur Gears

Pinion Gears

Internal Gears

**Spur gear geometries**

- **Pitch circle**: is the imaginary circle on which most gear calculations are made. When two gears meet their pitch circles are tangent to each other.
- **Pitch diameter ($D_p$) and pitch radius ($r$)**: These are the diameter and radius of the pitch circle.
- **Pitch point**: The point on the imaginary line joining the centers of the two meshing gears where the pitch circle touch.

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Figure 14.1  Spur gear drive.

Text Reference: Figure 14.1, page 316

Spur gear geometries

- **Addendum circle**: It is the circle that bounds the outer ends of the teeth and whose center is at the center of the gear (Fig. 7.2).
- **Dedendum circle**: It is the circle that bounds the bottoms of the teeth and whose center is at the center of the gear (Fig. 7.2).
- **Addendum (a)**: is the radial distance from the pitch circle to the outer end of the teeth. (Fig. 7.2).
- **Dedendum (b)**: is the radial distance from the pitch circle to the bottom of the teeth. (Fig. 7.2).

- **Circular pitch (Pc)**: is the distance between corresponding points on adjacent teeth measured along the pitch circle (Fig. 7.2).
- **Diametral pitch (Pdd)**: specifies the number of teeth per inch of pitch diameter.
- **Tooth space**: is the space between the adjacent teeth measured along the pitch circle (Fig. 7.2).
- **Tooth thickness**: is the thickness of the tooth measured along the pitch circle (Fig. 7.2).

- **Face width (W)**: is the length of the tooth measured parallel to the gear (Fig. 7.2).
- **Face**: is the surface between the pitch circle and the top of the tooth (Fig. 7.2).
- **Flank**: is the surface between the pitch circle and the bottom of the tooth (Fig. 7.2).
- **Pressure angle (φ)**: is the angle between the line of action and a line tangent to the two pitch circles at the pitch point. (Fig. 14.8 Hamrock).

- **Line of action**: is the locus of all the points of contact between two meshing teeth from the time the teeth go into contact until they lose contact.
- **Pinion**: is the smaller of the two meshing gears.
- **Backlash**: is the difference (clearance) between the tooth thickness of one gear and the tooth space of the meshing gear measured along the pitch circle (Fig. 7.5).

- **Clearance (c)**: is the addendum minus dedendum.
- **Working depth**: is the distance that one tooth of a meshing gear penetrates into the tooth space.
- **Base circle**: is an imaginary circle about which the tooth involute profile is developed.
- **Fillet**: is the radius that occurs where the flank of the tooth meets the dedendum circle.
- **Module**: replaces diametral pitch in metric system.
**Basic formulas for spur gears**

- Diametral pitch, \( P_d = \frac{N_p}{D_p} \)
- Circular pitch, \( P_c = \frac{D_c}{N_p} \)
- Addendum, \( a = \frac{1}{P_d} \)
- Dedendum, \( b = \frac{1.250}{P_d} \)

**Specifications for standard gear teeth**

<table>
<thead>
<tr>
<th>Item</th>
<th>Full depth &amp; pitches coarser than 20</th>
<th>Full depth &amp; pitches finer than 20</th>
<th>14.5” full depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure angle</td>
<td>20°</td>
<td>25°</td>
<td>20°</td>
</tr>
<tr>
<td>Addendum (in.)</td>
<td>1.0/ ( P_d )</td>
<td>1.0/ ( P_d )</td>
<td>1.0/ ( P_d )</td>
</tr>
<tr>
<td>Dedendum (in.)</td>
<td>1.250/ ( P_d )</td>
<td>1.250/ ( P_d )</td>
<td>1.2/ ( P_d ) + 0.002</td>
</tr>
</tbody>
</table>

**Basic formulas for spur gears**

- Clearance, \( c = b - a = 0.250 \frac{d}{P_d} \)
- Where
  - \( D_p \) = pitch diameter of pinion
  - \( N_p \) = number of teeth on the pinion
- It can be shown that \( P_d \times P_c = \pi \)

**Basic formulas for spur gears**

- Center to center distance
  - \( \text{CtoC} = \frac{D_{pp} + D_{pg}}{2} = \frac{N_p + N_g}{2 \ P_d} \)
  - Where
    - \( D_{pp} \) = pitch diameter of pinion
    - \( N_p \) = number of teeth on the pinion
    - \( D_{pg} \) = pitch diameter of gear
    - \( N_g \) = number of teeth on the gear
    - \( P_d \) = Diametral pitch

**Metric System**

- Module (m) = \( \frac{1}{P_d} \)
- See Table 11.1 for equivalents
- Normally they are not converted

**Metric System**

- Diametral pitch, \( P_d = \frac{1}{m} \)
- Circular pitch, \( P_c = \pi \ m \)
- Addendum, \( a = m \)
- Dedendum, \( b = 1.25 \ m \)
**Inch units**
- A spur gear of the 14 ½ degree involute system has 32 teeth of diametral pitch 8. Find
  - The pitch diameter
  - The circular pitch
  - The outside diameter (addendum diameter)

**Metric units**
- A spur gear of the 14 ½ degree involute system has a module of 8 mm and 35 teeth. Find
  - The pitch diameter
  - The circular pitch
  - The outside diameter (addendum diameter)

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**Example Problem 11-3: Pressure Angle**

- For the set of gears shown in Figure 11-17, if diametral pitch is 8, find the pitch diameter, circular pitch, and shaft center-to-center distance.
- The pinion has 16 teeth and the gear has 32 teeth.

**Example Problem 11-3: Pressure Angle (cont'd.)**

- Centerline distance:
  \[ C - C = \frac{D_1}{2} + \frac{D_2}{2} \quad \text{or} \quad C - C = \frac{N_1 + N_2}{2P_d} \]
  \[ C - C = \frac{16 + 32}{2 \times 8} \]
  \[ C - C = 3 \text{ inches} \]
To understand the gears one should be familiar with the gear terminology.

Spur gears are most commonly used for transmission of power.

Speed of mating gears is inversely proportional to the number of teeth.

Mating gears should have the same diametral pitch.

A number of gear manufacturing methods are available.

Good gear design should take care of the power, speed, life and material properties.