12. Spur Gear Design and Selection

Objectives

- Apply principles learned in Chapter 11 to actual design and selection of spur gear systems.
- Calculate forces on teeth of spur gears, including impact forces associated with velocity and clearances.
- Determine allowable force on gear teeth, including the factors necessary due to angle of module of tooth shape and materials selected for gears.
- Design actual gear systems, including specifying materials, manufacturing accuracy, and other factors necessary for complete spur gear design.
- Understand and determine necessary surface hardness of gears to minimize or prevent surface wear.
- Understand how lubrication can cushion the impact on gearing systems and cool them.
- Select standard gears available from stocking manufacturers or distributors.

Standard Proportions

- American Standard Association (ASA)
- American Gear Manufacturers Association (AGMA)
- Brown and Sharp
- 14 ½ deg; 20 deg; 25 deg pressure angle
- Full depth and stub tooth systems

Specifications for Standard Gear Teeth

<table>
<thead>
<tr>
<th>Item</th>
<th>Full depth &amp; pitches finer than 20</th>
<th>Full depth &amp; pitches finer than 20</th>
<th>14 ½º full depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>20º</td>
<td>25º</td>
<td>20º</td>
</tr>
<tr>
<td>Addendum</td>
<td>1.0/P</td>
<td>1.0/P</td>
<td>1.0/P</td>
</tr>
<tr>
<td>Dedendum</td>
<td>1.250/P</td>
<td>1.250/P</td>
<td>1.2/P + 0.002</td>
</tr>
</tbody>
</table>

Forces on Spur Gear Teeth

- $F_t = \text{Transmitted force}$
- $F_n = \text{Normal force or separating force}$
- $F_r = \text{Resultant force}$
- $\theta = \text{pressure angle}$
- $F_n = F_t \tan \theta$
- $F_r = \frac{F_t}{\cos \theta}$

Example Problem 12-1: Forces on Spur Gear Teeth

- 20-tooth, 8 pitch, 1-inch-wide, 20º pinion transmits 5 hp at 1725 rpm to a 60-tooth gear.
- Determine driving force, separating force, and maximum force that would act on mounting shafts.
Example Problem 12-1: Forces on Spur Gear Teeth

- 20-tooth, 8-pitch, 1-inch-wide, 20° pinion transmits 5 hp at 1725 rpm to a 60-tooth gear.
- Determine driving force, separating force, and maximum force that would act on mounting shafts.

Find pitch circle:

\[ \theta_p = \frac{N_p}{P_d} \]  

\[ \theta_p = \frac{20}{8} \text{ teeth/in} = 2.5 \text{ in} \]  

\[ \theta_p = \frac{20 \text{ teeth}}{8 \text{ in}} = 2.5 \text{ in} \]  

Example Problem 12-1: Forces on Spur Gear Teeth (cont’d.)

Find transmitted force:

\[ F_t = \frac{2T}{D_p} \]

\[ F_t = \frac{2(183 \text{ in-lb})}{2.5 \text{ in}} = 146 \text{ lb} \]

Find separating force:

\[ F_n = F_t \tan \theta \]

\[ F_n = 146 \text{ lb} \tan 20° \]

\[ F_n = 53 \text{ lb} \]

Find maximum force:

\[ F_r = F_t \cos \theta \]

\[ F_r = 146 \text{ lb} \cos 20° \]

\[ F_r = 155 \text{ lb} \]

Example Problem 12-2: Surface Speed

- In previous problem, determine the surface speed:

\[ V_m = \frac{\pi D_n}{12} \text{ ft/min} \]

\[ V_m = \frac{\pi D_n}{1000} \text{ m/min} \]

Surface Speed

- Surface speed \((V_m)\) is often referred to as pitch-line speed

\[ V_m = \frac{\pi D_p}{12} \text{ ft/min} \]

\[ V_m = \frac{\pi D_p}{1000} \text{ m/min} \]

Strength of Gear Teeth

- Lewis form factor method

Forces on Gear Tooth

**Lewis equation**

\[ F_s = S_d Y b P_d \]

- \( F_s \) = Allowable dynamic bending force (lb)
- \( S_d \) = Allowable stress (lb/in²). Use endurance limit and account for the fillet as the stress concentration factor
- \( b \) = Face width (in.)
- \( Y \) = Lewis form factor (Table 12.1)
- \( P_d \) = Diametral pitch

**Table 12.1 Lewis form factors (Y)**

<table>
<thead>
<tr>
<th>Number of Teeth</th>
<th>Full-depth involute</th>
<th>20° Full-depth</th>
<th>20° Involute</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0.716</td>
<td>0.710</td>
<td>0.711</td>
</tr>
<tr>
<td>13</td>
<td>0.693</td>
<td>0.687</td>
<td>0.688</td>
</tr>
<tr>
<td>14</td>
<td>0.668</td>
<td>0.661</td>
<td>0.662</td>
</tr>
<tr>
<td>16</td>
<td>0.643</td>
<td>0.636</td>
<td>0.633</td>
</tr>
<tr>
<td>18</td>
<td>0.620</td>
<td>0.613</td>
<td>0.610</td>
</tr>
<tr>
<td>20</td>
<td>0.598</td>
<td>0.591</td>
<td>0.588</td>
</tr>
<tr>
<td>22</td>
<td>0.578</td>
<td>0.570</td>
<td>0.567</td>
</tr>
<tr>
<td>24</td>
<td>0.558</td>
<td>0.550</td>
<td>0.547</td>
</tr>
<tr>
<td>26</td>
<td>0.539</td>
<td>0.531</td>
<td>0.527</td>
</tr>
<tr>
<td>28</td>
<td>0.521</td>
<td>0.513</td>
<td>0.509</td>
</tr>
<tr>
<td>30</td>
<td>0.504</td>
<td>0.496</td>
<td>0.492</td>
</tr>
<tr>
<td>32</td>
<td>0.489</td>
<td>0.481</td>
<td>0.477</td>
</tr>
<tr>
<td>34</td>
<td>0.475</td>
<td>0.467</td>
<td>0.463</td>
</tr>
<tr>
<td>36</td>
<td>0.462</td>
<td>0.454</td>
<td>0.450</td>
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<td>38</td>
<td>0.451</td>
<td>0.443</td>
<td>0.439</td>
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<tr>
<td>40</td>
<td>0.442</td>
<td>0.434</td>
<td>0.430</td>
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<td>0.433</td>
<td>0.425</td>
<td>0.421</td>
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<td>44</td>
<td>0.426</td>
<td>0.418</td>
<td>0.414</td>
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<tr>
<td>46</td>
<td>0.420</td>
<td>0.412</td>
<td>0.408</td>
</tr>
<tr>
<td>48</td>
<td>0.415</td>
<td>0.408</td>
<td>0.404</td>
</tr>
<tr>
<td>50</td>
<td>0.410</td>
<td>0.403</td>
<td>0.399</td>
</tr>
<tr>
<td>52</td>
<td>0.406</td>
<td>0.400</td>
<td>0.396</td>
</tr>
<tr>
<td>54</td>
<td>0.403</td>
<td>0.396</td>
<td>0.392</td>
</tr>
<tr>
<td>56</td>
<td>0.399</td>
<td>0.392</td>
<td>0.388</td>
</tr>
<tr>
<td>58</td>
<td>0.396</td>
<td>0.390</td>
<td>0.386</td>
</tr>
<tr>
<td>60</td>
<td>0.393</td>
<td>0.386</td>
<td>0.382</td>
</tr>
<tr>
<td>62</td>
<td>0.390</td>
<td>0.383</td>
<td>0.379</td>
</tr>
<tr>
<td>64</td>
<td>0.388</td>
<td>0.381</td>
<td>0.377</td>
</tr>
<tr>
<td>66</td>
<td>0.386</td>
<td>0.379</td>
<td>0.375</td>
</tr>
</tbody>
</table>

- Pinion: \( S_n = 0.5 S_u = 0.5 (95 \text{ ksi}) = 47.5 \text{ ksi} \) \((12-9)\)

\[ F_s = S_n b Y \]

- Find Lewis form factor \( Y \) from Table 12-1, assuming full-depth teeth:
  \[ Y = 0.320 \]

\[ F_s = 47,500 \left( \frac{1}{8} \right) 0.320 \]

\[ F_s = 1900 \text{ lb} \]

- In Example Problem 12-1, determine the force allowable \( F_s \) on these teeth if the pinion is made from an AISI 4140 annealed steel, the mating gear is made from AISI 1137 hot-rolled steel, and long life is desired.

- Gear: \( S_n = 0.5 S_u = 0.5 (38 \text{ ksi}) = 19 \text{ ksi} \) \((12-1)\)

\[ F_s = 1900 \text{ lb} \]

**Example Problem 12-3: Strength of Gear Teeth (cont’d.)**

- Gear: \( S_n = 0.5 S_u = 0.5 (88 \text{ ksi}) = 44 \text{ ksi} \) \((Table 12-1)\)

\[ Y = 0.421 \]

\[ F_s = 44,000 \left( \frac{1}{8} \right) 0.421 \]

\[ F_s = 2316 \text{ lb} \]

- Use \( F_s = 1900 \text{ lb} \) for design purposes.

**Classes of Gears**

- Transmitted load depends on the accuracy of the gears
- Gear Manufacture
  - Casting
  - Machining
    - Forming
    - Hobbing
    - Shaping and Planing
  - Forming
    - stamping

**Force Transmitted**

- Transmitted load depends on the accuracy of the gears
- A dynamic load factor is added to take care of this.

\[ F_t = \text{Transmitted force} \]

\[ F_d = \text{Dynamic force} \]

- Commercial
  \[ F_d = \frac{600 + V_m}{600} F_t \]
Classes of Gears

- Carefully cut: \[ F_d = \frac{1200 + V_m}{1200} F_t \]
- Precision: \[ F_d = \frac{78 + V_m^{0.5}}{78} F_t \]
- Hobbed or shaved: \[ F_d = \frac{50 + V_m^{0.5}}{50} F_t \]

Design Methods

- Strength of gear tooth should be greater than the dynamic force: \[ F_d \geq F_s \]
- You should also include the factor of safety: \[ \frac{F_d}{N_{sf}} \geq F_s \]

Table 12.2 Service Factors

<table>
<thead>
<tr>
<th>N_{sf}</th>
<th>Service Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 &lt; N_{sf} &lt; 1.25</td>
<td>Uniform load, without shock</td>
</tr>
<tr>
<td>1.25 &lt; N_{sf} &lt; 1.5</td>
<td>Medium shock, frequent starts</td>
</tr>
<tr>
<td>1.5 &lt; N_{sf} &lt; 1.75</td>
<td>Moderately heavy shock</td>
</tr>
<tr>
<td>N_{sf} &lt; 2</td>
<td>Heavy shock</td>
</tr>
</tbody>
</table>

Face width of Gears

- Relation between the width of gears and the diametral pitch: \[ \frac{P_d}{b} < \frac{12.5}{P_d} \]

Example Problem 12-4: Design Methods

If, in Example Problem 12-1, the gears are commercial grade, determine dynamic load and, based on force allowable from Example Problem 12-3, would this be an acceptable design if a factor of safety of 2 were desired?

- Use surface speed and force transmitted from Example Problems 12-2 and 12-3.

\[ F = \frac{V_m F_x}{600} \]

- Comparing to force allowable:

\[ \frac{P_d}{b} \geq F_d \]

\[ \frac{950 lb}{2} \geq 421 lb \]

\[ \frac{950 lb}{2} = 421 lb \]

Therefore:

\[ \frac{950 lb}{2} \geq 421 lb \]

Example Problem 12-5: Design Methods

- Spur gears from the catalog page shown in Figure 12-3 are made from a .2% carbon steel with no special heat treatment.
- What factor of safety do they appear to use in this catalog?
- Try a 24-tooth at 1800 rpm gear for example purposes.

\[ F_x = \frac{600}{V_m F_t} \]

\[ F_x = \frac{600}{1129} = 0.53 \text{~lb/in^2} \]

\[ S_u = 30 \text{~ksi} \]

\[ S_y = 55 \text{~ksi} \]

\[ S_y = 55 \text{~ksi} \]

\[ D_t = \frac{N_p}{P_D} \]

\[ D_t = 24 \text{~in} \]

\[ D_t = 24 \text{~in} \]
Example Problem 12-5: Design Methods (cont'd)

Find $V_m$:
\[
V_m = \frac{\pi D_p n}{12} \quad \text{in (1800 rpm) ft}
\]
\[
V_m = \frac{\pi \times 4 \text{ in} \times 1200 \text{ rpm}}{12 \text{ in}} = 942 \text{ ft/min}
\]

Find $F_s$:
\[
F_s = S_n b Y
\]
\[
P_{d} \quad \text{from Table 12-1}
\]
\[
Y = 0.302
\]
\[
F_s = 27,500 (\frac{3}{4}) \times 0.302
\]
\[
F_s = 519 \text{ lb}
\]

Example Problem 12-5: Design Methods (cont'd)

- Set $F_s = F_d$ and solve for $F_t$:
\[
F_d = F_s = \frac{(600 + V_m)}{600}
\]
\[
F_t = \frac{519 \text{ lb}}{600 + 942} \times 600
\]
\[
F_t = 202 \text{ lb}
\]

- $T = \frac{F_t D_p^2}{600}$
\[
T = \frac{202 \text{ lb} \times 2 \text{ in}^2}{600} = 202 \text{ in-lb}
\]

- $P = \frac{T n}{63,000}$
\[
P = \frac{202 \text{ in-lb} \times 1800}{63,000} = 5.8 \text{ hp}
\]

Example Problem 12-5: Design Methods (cont'd)

- Compared to catalog:
\[
N_{sf} = \frac{\text{hp - calculated}}{\text{hp - catalog}}
\]
\[
N_{sf} = \frac{5.8}{4.14} = 1.4
\]

Example Problem 12-5: Design Methods (cont'd)

- Appears to be reasonable value.
- Manufacturer may also have reduced its rating for wear purposes as these are not hardened gears.

Example Problem 12-6: Design Methods

- Pair of commercial-grade spur gears is to transmit 2 hp at a speed of 900 rpm of the pinion and 300 rpm for gear.
- If class 30 cast iron is to be used, specify a possible design for this problem.
- The following variables are unknown:
  - $P_d$
  - $D_p$
  - $b$
  - $N_t$
  - $N_{sf}$
  - $\theta$
- As it is impossible to solve for all simultaneously, try the following:
  - $P_d = 12$
  - $N_t = 48$
  - $\theta = 14.5^\circ$
  - $N_{sf} = 2$
- Solve for face width $b$.

- Surface speed:
\[
V_m = \frac{\pi D_p n}{12}
\]
\[
V_m = \frac{\pi \times 2 \text{ in} \times 900 \text{ rpm}}{12 \text{ in}} = 943 \text{ ft/min}
\]

Example Problem 12-6: Design Methods (cont'd.)

- Dynamic force
\[
F_d = \frac{600 + V_m}{600}
\]
\[
P_{d} \quad \text{from Table 12-1}
\]
\[
F_t = \frac{F_d}{N_{sf}}
\]
\[
F_t = \frac{(600 + 943)}{600} \times \frac{1}{2}
\]
\[
F_t = 170 \Delta
\]

- Since width $b$ is the unknown:
\[
F_t = \frac{S_n b Y}{P_{d}}
\]
\[
F_t = \frac{S_n b Y}{P_{d}}
\]

- Class 30 CI: $S_n = 30$ ksi; $S_m = 4.5$ ksi ($4$ is used because cast iron)
- $S_n = 12$ ksi
- $Y = 344$
Example Problem 12-6: Design Methods (cont'd.)

- Substituting:
  \[
  \frac{12000 \times 0.344}{2(0.2)} = 180
  \]
  \[
  b = 1.0 \text{ inches}
  \]
- Check ratio of width to pitch:
  \[
  \frac{8}{12} < b < 12.5
  \]
  \[
  \frac{8}{12} < 1 < 12.5
  \]
  \[
  0.66 < 1 < 1.04
  \]

• This is an acceptable design.
• Many other designs are also possible.

Example Problem 12-6: Design Methods (cont'd.)

• To increase the dynamic beam strength of the gear
  - Increase tooth size by decreasing the diametral pitch
  - Increase face width up to the pitch diameter of the pinion
  - Select material of greater endurance limit
  - Machine tooth profiles more precisely
  - Mount gears more precisely
  - Use proper lubricant and reduce contamination

Buckingham Method of Gear Design

- It offers greater flexibility
- Expected error is based on different-pitch teeth
- More conservative design

Buckingham Method of Gear Design

Table 12.3 Values of C for \( e = 0.001 \) inch

<table>
<thead>
<tr>
<th>Material</th>
<th>14 ½ deg</th>
<th>20 deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gray iron and Gray iron</td>
<td>800</td>
<td>830</td>
</tr>
<tr>
<td>Gray iron and steel</td>
<td>1,100</td>
<td>1,140</td>
</tr>
<tr>
<td>Steel and steel</td>
<td>1,600</td>
<td>1,660</td>
</tr>
</tbody>
</table>

Buckingham Method of Gear Design

\[
F_d = F_t + 0.05 V_m \left( bC + F_t \right) + 0.05 V_m \left( bC + F_t \right)^{0.5}
\]
Fig. 12.5 Recommended maximum error in gear teeth

**Example Problem 12-7: Buckingham Method of Gear Design and Expected Error**

- A pair of steel gears is made from annealed AISI 3140.
- Each gear has a surface hardness of BHN = 350.
- The pinion is a precision, 16 pitch, 20° involute, with 24 teeth 1 inch wide.
- The gear has 42 teeth.
- To transmit 3 hp at a speed of 3450 rpm for a safety factor of 1.4, is this a suitable design?

**Example Problem 12-7: Buckingham Method of Gear Design and Expected Error (cont'd.)**

- Find torque:
  \[ T = \frac{P}{63,000} \]
  \[ T = \frac{3}{63,000} = 55 \text{ in-lb} \]

- Find force transmitted:
  \[ F_t = \frac{2T}{D_p} \]
  \[ F_t = \frac{2 \times 55}{1.5} = 73 \text{ lb} \]

- Find surface speed:
  \[ V_m = \frac{\pi D_p n}{12} \]
  \[ V_m = \frac{\pi \times 1.5 \times 3450}{12} = 1355 \text{ ft/min} \]

- Find force allowable (\( F_s \)):
  \[ F_s = S_n b Y \]
  \[ Y = 0.337 \]
  \[ F_s = 47,500 \times 0.337 = 1000 \text{ lb} \]

- Expected error from Figure 12-4:
  \[ e = 0.0005 \]

- The value of \( C \) from Table 12-3 for steel and steel and 20° involute angle gears is 1660:
  \[ C = 1660 \]

- Solve for dynamic load using Buckingham’s equation:
  \[ F_d = F_t + 0.015 \frac{b N_p (D_p + D_g)}{D_p} \]

**Wear strength (Buckingham)**

\[ F_w = D_p b K_s \left( \frac{2 D_p}{D_p + D_g} \right) \]

- This meets the criteria.
Example Problem 12-8: Wear of Gears

- In prior example problem, verify the surface is suitable for wear considerations.
  For wear use $N_{ss} = 1.2$

  - Wear formula:
    \[ F_w = D_p \cdot b \cdot Q \cdot Kg \]

  - Find $Q$:
    \[ Q = \frac{2 \cdot N_g}{N_g + N_p} \]
    \[ Q = \frac{2 \cdot 42}{42 + 24} \]
    \[ Q = 1.27 \]

  - $K_g = 270$

  - In prior example problem, verify the surface is suitable for wear considerations.

Example Problem 12-8: Wear of Gears (cont'd.)

- Substituting into equation 12-16:
  \[ F_w = 1.5 \cdot (1.27 \cdot 270) \]
  \[ F_w = 514 \]

- This would not be suitable. Try if surfaces each had a BHN = 450.

  - $K_g = 470$
  \[ F_w = 1.5 \cdot (1.27 \cdot 470) \]
  \[ F_w = 895 \]
  \[ F_w > F_d \]
  \[ 895 > 699 \]
  \[ 748 > 699 \]

- This would now be acceptable if the gear teeth were hardened to a BHN of 450.