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Machine Design

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3. Stress and Deformation

Objectives

- Review the types of stresses caused from axial, bending, shear, and torsion loading.
- Review the relationship between stresses in the part and the strength or stress-carrying ability of the part, and begin to appreciate the relationship between the two.
- Distinguish between the ability of a material to carry loads in shear versus axial loading, and the relationship between these types of stresses.

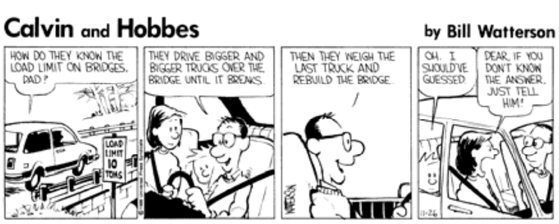
3. Stress and Deformation

Objectives

- Review the principles of deformation and whether those levels of deformation are acceptable to the design being analyzed.
- Review beam deflection formulas and their use in design problems.

Introduction to Failure Analysis

- Failure definition
  - A part fails whenever it no longer fulfils its required function



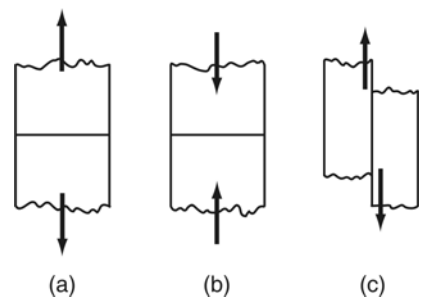
Failure Analysis

- Type of failures
  - Static loads
  - Dynamic loads – fatigue failure
- Modes of failure
  - Ductile – appreciable deformation
  - Brittle – relatively no deformation
  - Wear – due to friction
  - Creep – elevated temperatures

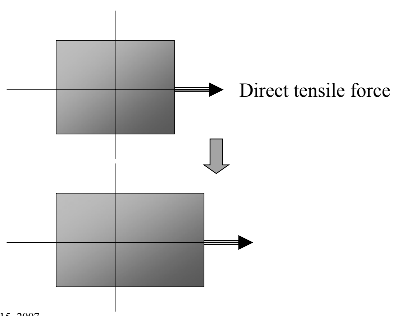
Modes of failure

- Stress
- Deformation
- Wear
- Corrosion
- Vibration
- Environmental damage
- Loosening of fastening devices

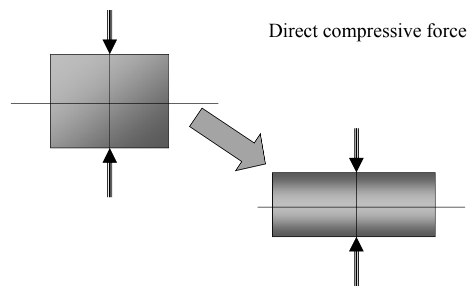
Fig. 3.2 Tension, Compression and Shear



Static Force



Static Force



Conditions for tension and compression

- Straight load carrying member
- Line of action passes through the centroid of the cross section of the member
- Member is of uniform cross section
- Material is homogeneous

Tensile Strength

$$\text{Stress, } S = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

$$\text{Strain, } \varepsilon = \frac{\text{Change in length}}{\text{Original length}}$$

Deformation

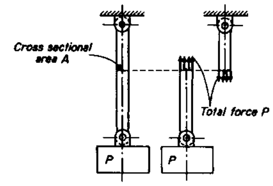
$$\text{Strain} = \frac{\text{Stress}}{\text{E}}$$

$$\text{Strain} = \frac{\text{Deformation}}{\text{Original length}}$$

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

$$\text{Deformation, } \delta = \frac{\text{Force} \times \text{Original length}}{\text{E} \times \text{Area}} = \frac{\text{F l}}{\text{E A}}$$

If P is equal to 5,000 lb and let the bar be 3 in wide and 0.5 in thick. The uniform bar is 60 in long and the material is steel. Find the stress in the uniform portion of the bar. Find the deformation of the uniform portion of the bar.
E = 30 x 10^6 psi



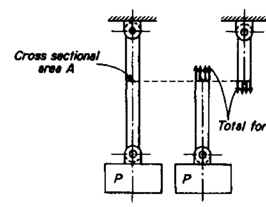
If P is equal to 5,000 lb and let the bar be 3 in wide and 0.5 in thick. The uniform bar is 60 in long and the material is steel. Find the stress in the uniform portion of the bar. Find the deformation of the uniform portion of the bar.

- Cross sectional area = $3 \times 0.5 = 1.5$ in^2
- Stress = $\frac{5000}{1.5}$ = 3,333 psi
- Deformation = $\frac{5000 \times 60}{1.5 \times 30{,}000{,}000}$

= 0.00667 in

If P is equal to 22,500 N and let the bar be 75 mm wide and 13 mm thick. The uniform bar is 1500 mm long and the material is steel. Find the stress in the uniform portion of the bar. Find the deformation of the uniform portion of the bar.

E = 206 900 MPa

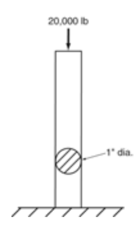


If P is equal to 22,500 N and let the bar be 75 mm wide and 13 mm thick. The uniform bar is 1500 mm long and the material is steel. Find the stress in the uniform portion of the bar. Find the deformation of the uniform portion of the bar.

- Cross sectional area = $75 \times 13 = 975$ mm^2
- Stress = $\frac{22{,}500}{975}$ = 23.08 MPa
- Deformation = $\frac{22{,}500 \times 1{,}500}{975 \times 206{,}900}$

= 0.167 mm

Example Problem 3-1—Tensile and Compressive Axial Loads



- 20,000-pound load is applied to a one-inch diameter steel bar that is made from AISI 1020 hot-rolled steel with S_u = 55,000 psi and S_y = 30,000 psi.

a. What is the stress in the bar?

b. By how much did the bar shorten?

c. Will this bar return to its original length when the load is removed?

$$S = \frac{F}{A} = \frac{2{,}000 \text{ lb}}{\frac{\pi (1 \text{ in})^2}{4}} = 25{,}465 \text{ lb/in}^2 \quad (3\text{-}1)$$

Example Problem 3-1—Tensile and Compressive Axial Loads (cont'd.)

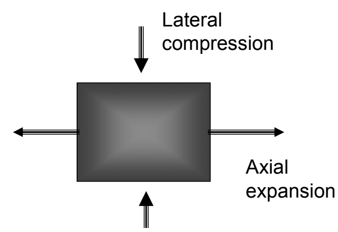
b. By how much did the bar shorten?

$$\delta = -\frac{FL}{AE} = \frac{20{,}000\ lb\ \ 6\ in}{\frac{\pi(1\ in)^2}{4}\ 30x10^6\ lb/in^2} = -.005\ in \qquad (3\text{-}2)$$

c. Will this bar return to its original length when the load is removed?

- The stress of 25,465 psi is less than S_y of 30,000 psi, so it will return to its original size as the yield limit was not exceeded.

Poisson's Ratio

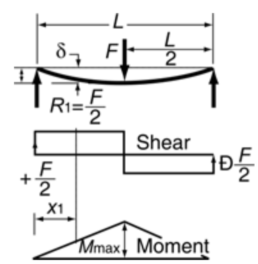


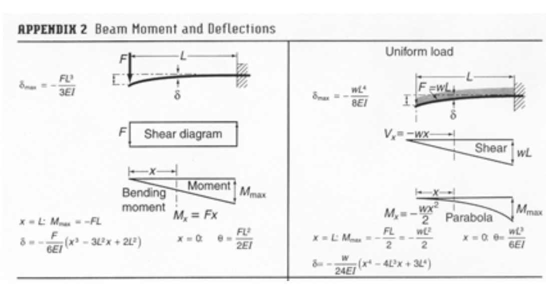
Poisson's Ratio

- Axial strain = $\frac{\text{Axial Deformation}}{\text{Axial dimension}}$
- Lateral strain = $\frac{\text{Lateral Deformation}}{\text{Lateral dimension}}$
- Poisson's ratio = $\frac{\text{Lateral Strain}}{\text{Axial Strain}}$

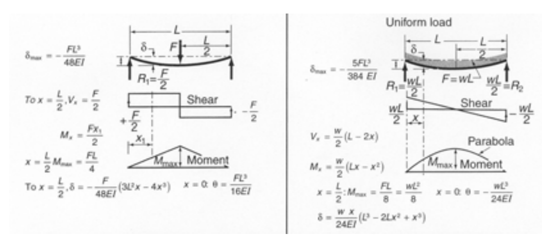
3.4 Stresses and Deflections due to Bending

- Shear force
- Bending moment
- Section Modulus, Z = $\frac{\text{Moment of Inertia}}{\text{Distance from the neutral axis}}$
- Stress, S = $\frac{M}{Z}$

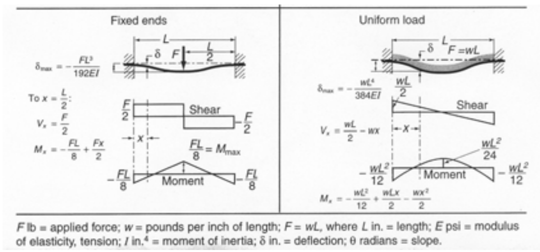




Cantilever Beam



Simply supported Beam



Fixed fixed Beam

Stresses and Deflections due to Bending

- Beam must be primarily in pure bending (no axial and shear loads)
- Beam is not subjected to torsion
- Beam material has the same modulus of elasticity in tension and compression
- Beam is not subjected to localized buckling

Bending moment

- Reference is to a beam, assumed for convenience to be horizontal and loaded and supported by forces, all of which lie in a vertical plane.
- The *bending moment* at any section of the beam is the moment of all forces that act on the beam to the left of that section, taken about the horizontal axis of the section.

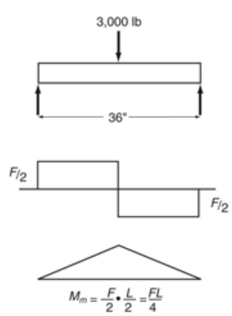
Bending moment

- The bending moment is positive when clockwise and negative when counterclockwise; a positive bending moment therefore bends the beam so that it is concave upward, and a negative bending moment bends it so that it is concave downward.

Example Problem 3-2—Stresses and Deflection Due to Bending

- For the 2"x 2" simply supported beam made from the same 1020 steel as Example Problem 3-1, assume a safety factor of 2 based on the ultimate stress allowable.
- a. What is the maximum stress in the beam?
- b. What is the maximum deflection in this beam?

c. Will this bar return to its original shape when this load is removed?
- d. For a safety factor of 2, based on ultimate, is this an acceptable design?



Example Problem 3-2—Stresses and Deflection Due to Bending (cont'd.)

a. What is the maximum stress in the beam?

$$I = \frac{bh^3}{12} = \frac{2\,in\,(2\,in)^3}{12} = 1.33\,in^4$$ (Appendix 3)

$$M_m = \frac{FL}{4}$$ (Appendix 2)

$$S = \frac{Mc}{I} = \frac{(FL)\,c}{(4)\,I}$$ (3-3)

$$S = 3{,}000\,lb\;\frac{36\,in\,1\,in}{4\,(1.33\,in)^4} = 20{,}300\;\;lb/in^2$$

Example Problem 3-2—Stresses and Deflection Due to Bending (cont'd.)

b. What is the maximum deflection in this beam?

$$\delta = \frac{F L^3}{-48\, E I}$$ (Appendix 2)

$$\delta = \frac{3000 lb (36\ in)^3}{48\ (30x10^6)\ lb/in^2 1.33 in^4} = -.073\ in$$

c. Will this bar return to its original shape when this load is removed?

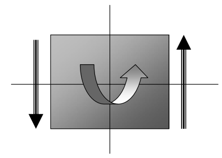
- Yes, the stress of 20,300 lb/in^2 is less than the yield of S_y= 30,000 lb/in^2.

d. For a safety factor of 2, based on ultimate, is this an acceptable design?

$$\frac{S_u}{N} = \frac{55{,}000 lb/in^2}{2} = 27{,}500\ lb/in^2$$

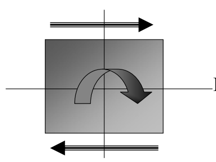
- It is acceptable as this is less than the actual stress of 20,300 lb/in^2.

Shear Force



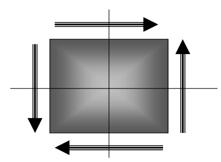
Negative – Counter clockwise

3.5 Shear Force



Positive – Clockwise

Shear force sign convention



Positive – Clockwise

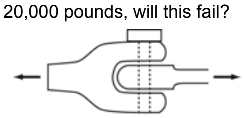
Negative – Counter clockwise

Direct shear stress, S_S

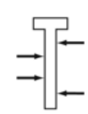
- Force tends to cut through a member
  - Punch and Die
  - Shear on a key in a shaft
- Similar to calculating direct tensile stress
- S_S = Shear force / Area in shear

Example Problem 3-3—Shear Stresses

- The yoke shown has a ½-inch diameter pin made from AISI 1040 cold drawn steel.
- For a load of 20,000 pounds, will this fail?



Yoke assembly



Free body diagram of pin

Example Problem 3-3—Shear Stresses (cont'd.)

(Appendix 4)

$$S_u = 80 \text{ KSI} \quad S_y = 71 \text{ KSI}$$

(3-4)

$$S_s = \frac{F}{A_s}$$

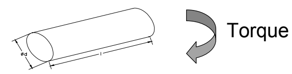
Double shear

$$S_s = \frac{F}{\frac{(2)\pi D^2}{4}}$$

$$S_s = \frac{20{,}000 \text{ lb}}{\frac{(2)\ \pi (.5 \text{ in})^2}{4}} = 50{,}930 \text{ lb/in}^2$$

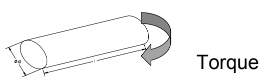
- Comparing to .5 or .6 of S_u, this stress is too high and the pin would fail.

3.6 Torsional Shear Stresses



- Torsional shear stress, $S_S = \frac{T\ c}{J}$
- J = Polar moment of inertia = $\frac{\pi \times d^4}{32}$
- c = radius of the shaft
- T = Torque
- d = diameter of shaft

Shear Stress in a shaft

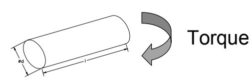


- Shear stress, $S_S = \frac{16 \times T}{\pi \times d^3}$

Where

- T = torque
- d = diameter of the shaft

Angular Deformation in a shaft



- Angular twist, $\theta = \frac{T \times l}{J \times G}$

Where

- T = torque
- l = length of the shaft
- G = Modulus of rigidity of shaft material
- J = Polar moment of inertia of the shaft $= \frac{\pi \times d^4}{32}$

Example Problem 3-4—Torsional Shear Stresses

- A round shaft transmits power delivered by a 10-hp electric motor turning at 1,750 rpm. If the shaft is made from AISI 1117 hot-rolled steel, ½ inch in diameter, and 12 inches in length:

  a. What is the stress in this shaft?

  b. What would the angular deflection of this shaft be?

  c. If a safety factor of 3 is desired, based on ultimate strength,
  how large should this shaft be?

Example Problem 3-4—Torsional Shear Stresses

- A round shaft transmits power delivered by a 10-hp electric motor turning at 1,750 rpm.
- If the shaft is made from AISI 1117 hot-rolled steel, ½ inch in diameter, and 12 inches in length:

  a. What is the stress in this shaft?

(Appendix 4)

$$S_u = 62 \text{ KSI} \qquad S_y = 34 \text{ KSI}$$

(2-6)

$$T = \frac{63{,}000 \text{ hp}}{\text{rpm}}$$

$$T = \frac{63{,}000\ (10)}{1750} = 360 \text{ in-lb}$$

(3-6)

$$S_s = \frac{T}{Z'}$$

(Appendix 3)

$$Z' = \frac{\pi D^3}{16}$$

$$Z' = \frac{\pi (.5 \text{ in})^3}{16} = .025 \text{ in}^3$$

$$S_s = \frac{T}{Z'} = \frac{360 \text{ in-lb}}{.025 \text{ in}^3} = 14{,}400 \text{ lb/in}^2$$

Example Problem 3-4—Torsional Shear Stresses (cont'd.)

b. What would the angular deflection of this shaft be?

(3-7)

$$\theta = \frac{T\,L}{J\,G}$$

(Appendix 3)

$$J = \frac{\pi D^4}{32} = \frac{\pi (.5\ in)^4}{32}$$

$$J = .0061\ in^4$$

$$\theta = \frac{T\,L}{J\,G} = \frac{360\ in\text{-}lb\ \ 12\ in}{.0061\ in^4\ \ 11.5 \times 10^6\ lb/in^2}$$

$$\theta = .062\ radians$$

Example Problem 3-4—Torsional Shear Stresses (cont'd.)

c. If a safety factor of 3 is desired, based on ultimate strength, how large should this shaft be?

$$\frac{(.5)\,Su}{(N)} = \frac{T}{Z'} = \frac{T}{\frac{\pi\,D^3}{16}}$$

$$D^3 = \frac{(N)\,16T}{.5\,S_u\,\pi}$$

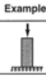
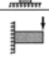
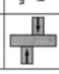
$$D = \left(\frac{(N)\,16T}{.5\,S_u\,\pi}\right)^{1/3}$$

$$D = \left(\frac{3\,(16)\,360\,in-lb}{(.5)\,62{,}000\,lb/in^2\,\pi}\right)^{1/3}$$

$$D = .562\ in$$

APPENDIX 1

Categorization of Stress Types (Basic Stress Theory)

Basic stress theory					
Method of molecule separation	**Type**	**Example**	**Stress**	**Deflection**	**Stress allowable**
Push together or pull apart	Axial		$S = \frac{F}{A}$	$\delta = \frac{FL}{AE}$	S_y or S_u
	Bending		$S = \frac{MC}{I}$ or $\frac{M}{Z}$ I or Z from App. 3 Calculate M from App. 2	δ from App. 2	S_y or S_u
Slide by	Torsion		$S = \frac{TC}{J}$ or $\frac{T}{Z'}$ J or Z' from App. 3	$\theta = \frac{TL}{JG}$	S_s if known; otherwise use .5 or .6 S_y or S_u
	Shear		$S = \frac{F}{A}$ $S = \frac{VQ}{IB}$	Usually don't care!	S_s if known; otherwise use .5 or .6 S_y or S_u