330:148 (g)
Machine Design

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4. Combined Stress and Failure Theories

- When parts have multiple types of loading or more than one type of stress from a single load

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Objectives

- Group stresses by type, separating the stresses into bending and axial versus shear and torsional stresses.
- Combine like types of stresses in an appropriate manner.
- Combine different types of stresses, using appropriate combined stress theories.
- Gain further understanding into how these combined stresses should be compared to the stress allowables for the materials being used in the design.

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Grouping Types of Stresses

- Axial and Bending
- Torsion and Shear

Appendix 1
p 466

Basic stress theory

Method of molecule separation	Type	Example	Stress	Deflection	Stress allowable
Push together or pull apart	Axial		$S = \frac{F}{A}$	$\delta = \frac{FL}{AE}$	S_y or S_u
	Bending		$S = \frac{MC}{I}$ or $\frac{M}{Z}$ I or Z from App. 3 Calculate M from App. 2	δ from App. 2	S_y or S_u
Slide by	Torsion		$S = \frac{TC}{J}$ or $\frac{T}{Z'}$ J or Z' from App. 3	$\theta = \frac{TL}{JG}$	S_{s} if known; otherwise use .5 or .6 S_y or S_u
	Shear		$S = \frac{F}{A}$ $S = \frac{VQ}{IB}$	Usually don't care!	S_{s} if known; otherwise use .5 or .6 S_y or S_u

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Combined Axial and Bending stresses

- Summation of stresses taking the directions into account
  - Column with an eccentric load
- $S = \pm S_{axial} \pm S_{bending} = \pm \frac{F}{A} \pm \frac{M}{Z}$
  - Tensile +
  - Compressive -

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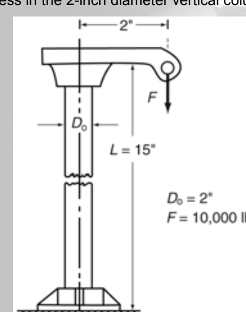
Example Problem 4-1:
Design of a Short Column with Eccentric Load

- Determine the stress in the 2-inch diameter vertical column shown.



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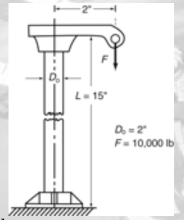

- First, determine stresses.
  - Axial stress

$$S = \frac{F}{A} = \frac{10{,}000\ lb}{\frac{\pi (2\ in)^2}{4}} = 3183\ \frac{lb}{in^2}$$

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Example Problem 4-1:
Design a Short Column with Eccentric Load (cont'd.)

– Bending stress:

$$S = \frac{M}{Z} = \frac{Fe}{\frac{\pi D^4}{32}} = \frac{10{,}000 lb(2in)}{\frac{\pi (2\ in)^4}{32}} = 25{,}460\ lb/in^2$$

$$S = \pm S_{axial} \pm S_{bending} \qquad (4\text{-}1)$$

$$S = -3183\ lb/in^2 - 25{,}460\ lb/in^2$$

$$S = 28{,}647\ lb/in^2$$

- The bending stress and axial stress add on the inner side of the column.
- Stresses subtract on the outer side so we are primarily concerned about the inner surface.

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Coplanar Shear Stresses

- Use vectorial addition

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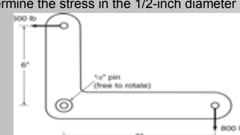
Example Problem 4-2: Coplanar Shear

- Determine the stress in the 1/2-inch diameter pin.



- As the bell crank is free to rotate, both forces create shear stresses in the pin.
  - Adding forces vectorially:

$$F_t = \sqrt{(F_1^2 + F_2^2)} \qquad \text{(perpendicular forces)}$$

$$F_t = \sqrt{(600\ lb)^2 + (800\ lb)^2}$$

$$F_t = 1000\ lb$$

$$S = \frac{F}{A} = \frac{1000\ lb}{\frac{\pi (1/2\ in)^2}{4}} = 5093\ lb/in^2$$

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Shear and Torsion

- Use vectorial addition since the stresses are of the same type
- S = $S_{torsion}$ + S_{shear}

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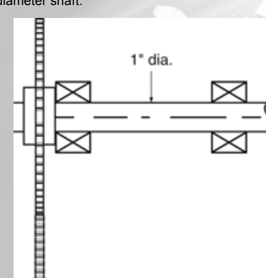
Example Problem 4-3: Combined Torsion and Shear

- A roller chain system transmits 50 hp at a speed of 300 rpm.
- If the chain sprocket has an effective pitch diameter of 10 inches, calculate the combined stress in the 1-inch diameter shaft.



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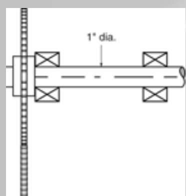


(2-6)

$$T = \frac{63{,}000\ (hp)}{n}$$

$$T = \frac{63{,}000\ (50\ hp)}{300\ rpm}$$

$$T = 10{,}500\ in\text{-}lb$$

Example Problem 4-3: Combined Torsion and Shear (cont'd.)

- Driving force in chain:

(2-5)

$$F = \frac{T}{r} = \frac{10{,}500\ in\text{-}lb}{\frac{10\ in}{2}} \qquad as:\ T = Fr$$

$$F = 2100\ lb$$

- Calculate shear and torsional stresses.
  - Shear in shaft:

$$S_s = \frac{F}{A} = \frac{2100\ lb}{\frac{\pi\,(1\ in)^2}{4}}$$

$$S_s = 2674\ lb/in^2$$

Example Problem 4-3: Combined Torsion and Shear (cont'd)

- Torsional stress:

$$S_s = \frac{T}{Z'}$$

(Appendix 3)

$$Z' = \frac{\pi D^3}{16}$$

$$Z' = \frac{\pi\,(1\ in)^3}{16} = .196\ in^3$$

$$S_s = \frac{10{,}500\ in\text{-}lb}{.196\ in^3}$$

$$S_s = 53{,}570\ lb/in^2$$

(4-3)

$$S_{total} = S_{torsion} + S_{shear}$$

$$S_{total} = 53{,}570\ lb/in^2 + 2{,}674\ lb/in^2$$

$$S_{total} = 56{,}250\ lb/in^2$$

- This value would be compared to shear stress allowable for the shaft material.

Normal and Shear Stresses

- Mohr's Circle
- σ = equivalent combined normal stress
- S = normal stress from bending or axial loads
- S_S = shear or torsional stress

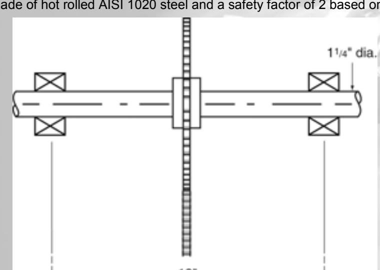
$$\sigma = \frac{S}{2} \pm \left[S_S^{\,2} + \left(\frac{S}{2}\right)^2 \right]^{1/2}$$

Normal and Shear Stresses

- S = F / A for axial loads
- S = M / Z for bending loads
- S_S = F / A for shear loads
- S_S = T c / J for torsion shear

Example Problem 4-4: Combined Normal and Shear Stress

- A center mounted chain drive system transmits 20 hp at a speed of 500 rpm.
- If the sprocket has a pitch diameter of 8 inches, would this be an acceptable design if the shaft is made of hot rolled AISI 1020 steel and a safety factor of 2 based on yield is desired?



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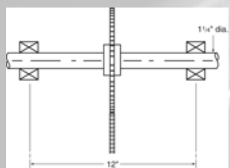


– Torque on shaft:

$$T = \frac{63{,}000\,hp}{n} \quad (2\text{-}6)$$

$$T = \frac{63{,}000\,(20\,hp)}{500\,rpm}$$

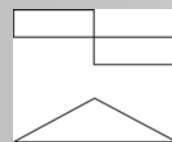
$$T = 2{,}520\,in-lb$$

Example Problem 4-4: Combined Normal and Shear Stress (cont'd.)

- Force in chain:

$$F = \frac{T}{r} = \frac{2520\ in\text{-}lb}{\frac{8\ in}{2}} = 630\ lb$$

– Bending moment in shaft:



(Appendix 2)

$$M_m = \frac{FL}{4}$$

$$M_m = \frac{630\ lb\ \ 12\ in}{4}$$

$$M_m = 1{,}890\ in\text{-}lb$$

Example Problem 4-4: Combined Normal and Shear Stress (cont'd.)

- Calculating torsional and bending stresses
  - Torsional shear stress:

$$S_s = \frac{T}{Z'}$$

(Z' from Appendix 3)

$$S_s = \frac{2520\ in\text{-}lb}{\frac{\pi D^3}{16}}$$

$$S_s = \frac{2520\ in\text{-}lb}{\frac{\pi (1.25\ in)^3}{16}}$$

$$S_s = 6570\ in\text{-}lb$$

  - Bending (normal) stress:

$$S = \frac{M}{Z} = \frac{1890\ in\text{-}lb}{\frac{\pi D^3}{32}}$$

(Z from Appendix 3)

$$S = \frac{1890\ in\text{-}lb}{\frac{\pi (1.25\ in)^3}{32}}$$

$$S = 9860\ lb/in^2$$

Example Problem 4-4: Combined Normal and Shear Stress (cont'd.)

- Combining stresses:

(4-4)

$$\sigma = \frac{S}{2} \pm \left(S_s^2 + \left(\frac{S}{2}\right)^2 \right)^{1/2}$$

$$\sigma = \frac{9860\ lb/in^2}{2} + \left((6570\ lb/in^2)^2 + \left(\frac{9860\ lb/in^2}{2}\right)^2 \right)^{1/2}$$

$$\sigma = 13{,}150\ lb/in^2$$

(Appendix 4)

$$S_y = 30{,}000\ lb/in^2$$

$$\frac{S_y}{2} = 15{,}000\ lb/in^2$$

- This is greater than the combined stress of 13,150 lb/in^2, so it is acceptable.

Combined maximum shear stress

- τ = Maximum combined shear stress
- S = normal stress
- S_S = shear stress

$$\tau = \left[S_S^{\,2} + \left(\frac{S}{2}\right)^2 \right]^{1/2}$$

Distortion energy theory

- Also called as von Mises theory
- Closely duplicate the failure of ductile materials under static, repeated and combined stresses
- To use this determine the two principal stresses using Mohr's circle or other means

$$\sigma' = \sqrt{\sigma_1^{\,2} + \sigma_2^{\,2} - \sigma_1\,\sigma_2}$$