330:148 (g) Machine Design

Nageswara Rao Posinasetti

5. Repeated Loading

Objectives

- Identify the various kinds of loading encountered on a part and learn to combine them as appropriate.
- Determine the level of stress in a material at which a hypothesized defect would not propagate.
- Recognize what types of factors affect this endurance limit for the material.
- Factor in the effect of shapes and discontinuities as they affect the stress concentration factors.
- Gain experience using fatigue equations when designing parts subject to repeated loads.

Dynamic Loads

- Forces that vary frequently in magnitude and direction are called dynamic loads.
- Varying load:
  - Magnitude changes but not the direction
- Reversing load:
  - Both magnitude and direction
- Shock load:
  - Impact

Dynamic Strength

- Fatigue failure in a material is when it fails even when the stress may not go beyond the proportional limit.

Dynamic Strength

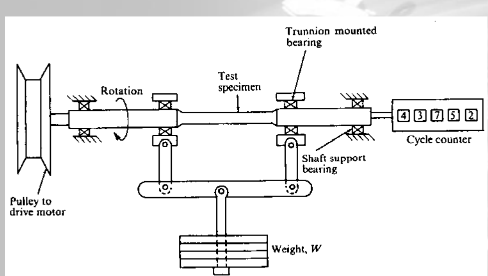


FIGURE 2.9 Fatigue testing machine

Courtesy Esposito - Machine Design, 1991, Delmar Publishers

Dynamic Strength

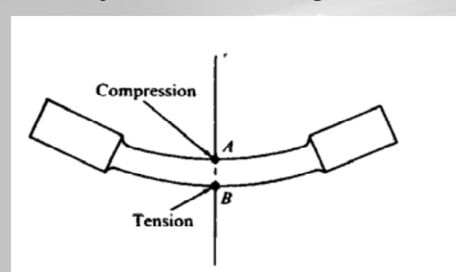
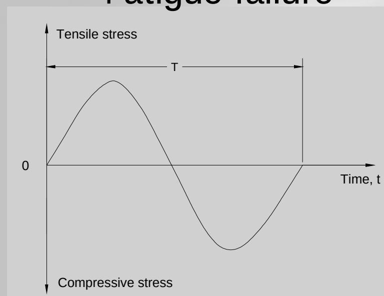


FIGURE 2.11 Shaft bending deformation

Courtesy Esposito - Machine Design, 1991, Delmar Publishers

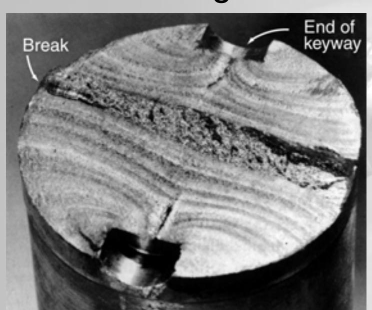
Fatigue failure



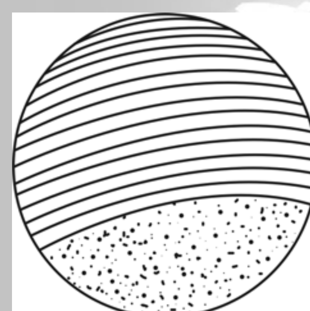
Fatigue failure

- After some number of stress reversal cycles a crack is initiated.
- This crack propagates towards the centre.
- Many more cracks are formed around the periphery.
- Ultimately the shaft breaks when the stress in the remaining solid area exceeds the ultimate strength, as shown in Fig. 5.4.

Fatigue failure



Fatigue failure



Endurance Limit and Endurance Strength

- It is the strength of a material to resist the propagation of cracks under stress reversals.
- Endurance Limit (S_n): Is the stress value below which an infinite number of cycles will not cause failure

Endurance Limit

$S_n = 0.5\ S_u$

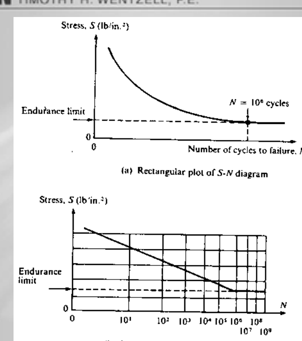


FIGURE 2.14 Typical endurance limit diagram

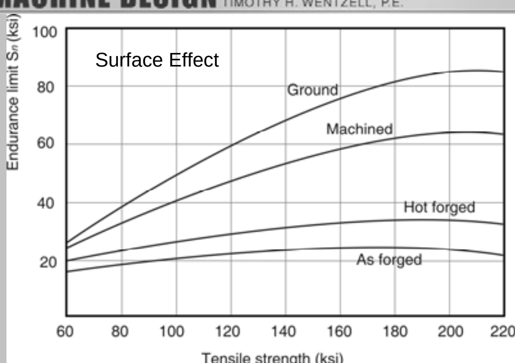
Courtesy Esposito - Machine Design, 1991, Delmar Publishers

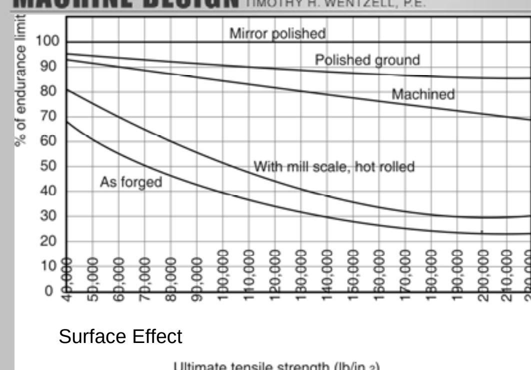
Modifying factors for endurance limits

- The actual value of endurance limit to be applied needs to be modified depending upon the application
- Type of stress
  - C_{type} = 0.8 axial
  - C_{type} = 0.8 shear or torsion
  - C_{type} = 1.0 bending

Modifying factors for endurance limits

- Size
  - C_{size} = 1.00 for < ½ inch
  - C_{size} = 0.85 for < ½ to 2 inches
  - C_{size} = 0.75 for < 2 inches and over
- Surface
  - Rough surfaces have stress raisers
- $S_n = C_{type} \times C_{size} \times C_{surface} \times S_n'$



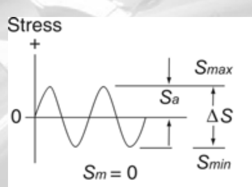


Variation of Stresses

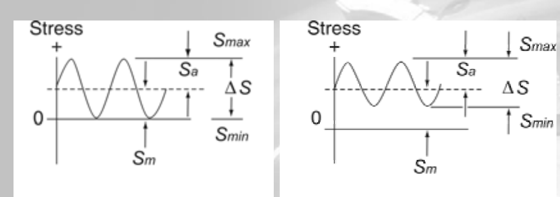
- Mean stress, S_m
- Alternating stress, S_a

$$S_m = \frac{S_{max} + S_{min}}{2}$$

$$S_a = \frac{S_{max} - S_{min}}{2}$$



(a)



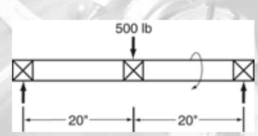
(b) (c)

Theories of failure

- Soderberg equation $\frac{1}{N} = \frac{S_m}{S_y} + \frac{S_a}{S_n}$

Example Problem 5-1— Theories of Failure: Soderberg Equation

- For a smooth rotating shaft with no sharp corners or change in shape, determine the required diameter under the loading condition shown.
- Ignore any torque.
- The surface of the shaft is highly polished.
- The shaft is made from annealed AISI 4140 steel.
- Use a safety factor of 2.



Example Problem 5-1— Theories of Failure: Soderberg Equation (cont'd.)

- For the first trial, assume the resultant diameter will be between ½ and 2 inches:

$S_u = 95$ ksi $S_y = 60$ ksi (Appendix 4)

- First calculate S_{max} and S_{min}:

(Appendix 2)

$$M_{max} = \frac{FL}{4}$$

$$M_{max} = \frac{500\ lb\ 40\ in}{4} = 5000\ in-lb$$

$$M_{min} = \frac{FL}{4} = -5000\ in-lb$$

$$S_{max} = \frac{M_{max}}{Z} = \frac{5000\ in-lb}{Z}$$

$$S_{min} = \frac{M_{min}}{Z} = -\frac{5000\ in-lb}{Z}$$

Example Problem 5-1—Theories of Failure: Soderberg Equation (cont'd.)

- Find S_{mean} and $S_{alternating}$:

(5-2)

$$S_{mean} = \frac{S_{max} + S_{min}}{2}$$

$$S_{mean} = \frac{\frac{5000\,in-lb}{Z} - \frac{5000\,lb-in}{Z}}{2} = 0$$

(5-3)

$$S_{alt} = \frac{S_{max} - S_{min}}{2}$$

$$S_{alt} = \frac{\frac{5000\ in-lb}{Z} - \frac{5000\ in-lb}{Z}}{2} = \frac{5000\ in-lb}{Z}$$

Example Problem 5-1—Theories of Failure: Soderberg Equation (cont'd.)

- Find the endurance limit modifying factors:

$S_n = C_{size}\ C_{surface}\ C_{type}\ S_n'$ (5-1)

- As surface is polished: $C_{surface} = 1$ (Figure 5-7b)

- As S_n' is unknown, use .5 S_u:

Use $C_{size} = .85$ assuming .5 < D < 2

Bending $C_{type} = 1$

$S_n = .85\ (1)\ (1)\ (.5)\ 95$ ksi

$S_n = 40.375$ ksi

Example Problem 5-1—Theories of Failure: Soderberg Equation (cont'd.)

- Substituting into the Soderberg equation:

$$\frac{1}{N} = \frac{S_m}{S_y} + \frac{S_a}{S_n}$$ (5-5)

$$\frac{1}{2} = \frac{\frac{5000\ in\text{-}lb}{Z}}{40{,}375\ lb/in^2}$$

$$Z = .247\ in^3$$

$$Z = \frac{\pi D^3}{32}$$ Appendix 3

$$D^3 = \frac{32\ Z}{\pi}$$

$$D = \sqrt[3]{\frac{32\ (.247\ in^3)}{\pi}}$$

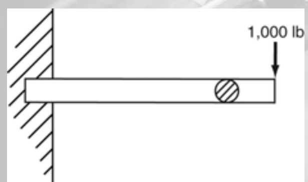
$$D = 1.36\ in$$

Use $D = 1\text{-}3/8$ or $1.375\ in$

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Example Problem 5-2—Theories of Failure: Soderberg Equation

- A 12-inch-long round cantilever beam is loaded repeatedly but no load reversal occurs.
- If the bar is made from annealed AISI 302 stainless steel and the surface is polished for a safety factor of 1.6, find the required diameter using the Soderberg method.



MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Example Problem 5-2—Theories of Failure: Soderberg Equation (cont'd.)

S_u = 90 ksi S_y = 37 ksi S_n = 34 ksi (Appendix 8)

- Find S_{max} and S_{min}:

(Appendix 2)

$$M_{max} = FL$$
$$M_{max} = 1000\ lb\ 12\ in$$
$$M_{max} = 12{,}000\ in-lb$$
$$M_{min} = 0$$
$$S_{max} = \frac{M_{max}}{Z} = \frac{12{,}000\ in-lb}{Z}$$
$$S_{min} = 0$$

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Example Problem 5-2—Theories of Failure: Soderberg Equation (cont'd.)

- Find S_{mean} and $S_{alternating}$:

(5-2)

$$S_{mean} = \frac{S_{max} + S_{min}}{2}$$
$$S_{mean} = \frac{\frac{12{,}000\ in\text{-}lb}{Z} + 0}{2}$$
$$S_{mean} = \frac{6{,}000\ in\text{-}lb}{Z}$$

(5-3)

$$S_{alt} = \frac{S_{max} - S_{min}}{2}$$
$$S_{alt} = \frac{\frac{12{,}000\ in\text{-}lb}{Z} - 0}{2}$$
$$S_{alt} = \frac{6{,}000\ in\text{-}lb}{Z}$$

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Example Problem 5-2—Theories of Failure: Soderberg Equation (cont'd.)

- Find the endurance limit modifying factors:

C_{size} = .85 (assume ½ < D < 2 inches)

C_{type} = 1 (bending)

$C_{surface}$ = 1 (polished surface)

$$S_n = C_{size}\ C_{surface}\ C_{type}\ S_n' \quad (5\text{-}1)$$
$$S_n = .85\ (1)\ (1)\ 34\ ksi$$
$$S_n = 28.9\ ksi$$

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Example Problem 5-2—Theories of Failure: Soderberg Equation (cont'd.)

- Substituting into the Soderberg equation:

(5-4)

$$\frac{1}{N} = \frac{S_m}{S_y} + \frac{S_a}{S_n}$$
$$\frac{1}{1.6} = \frac{\frac{6{,}000\ in\text{-}lb}{Z}}{37{,}000\ lb/in^2} = \frac{\frac{6{,}000\ in\text{-}lb}{Z}}{28{,}900\ lb/in^2}$$
$$Z = .592\ in^3$$

(Appendix 3)

$$Z = \frac{\pi D^3}{32}$$
$$D^3 = \frac{32\ Z}{\pi}$$
$$D = \sqrt[3]{\frac{32\ (.592\ in^3)}{\pi}}$$
$$D = 1.82\ inches$$

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Stress Concentration: Discontinuities in part contour

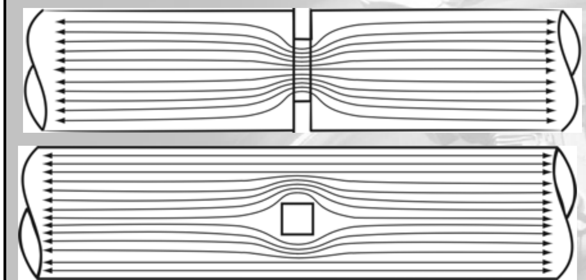
- Shafts with shoulders to accommodate the seating of bearings
- Key ways in shafts that use key to secure pulleys, cams and gears
- Threads on one end of a bolt and a head on the other end
- Fillet at the base of gear teeth

Stress Concentration

- Physical discontinuity is called a stress raiser or a region of stress concentration.

Stress concentration factor, K = $\dfrac{S_{max\_actual}}{S_{avg\_calc}}$

Stress Concentration



Stress concentration

$$S_{avg} = \frac{F}{A_B - A_H}$$

$$S_{max} = S_{avg} \times K$$

K = Stress concentration factor

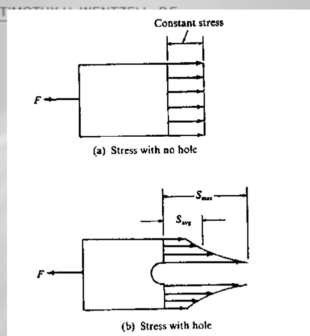


FIGURE 2.16 Stress distribution in uniform bar

Courtesy Esposito - Machine Design, 1991, Delmar Publishers

Stress concentration

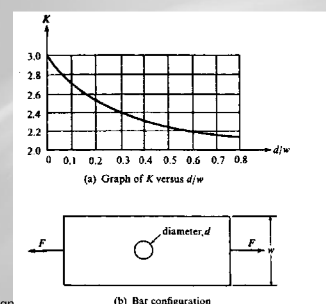
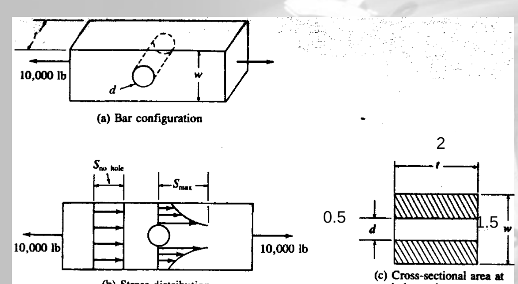


FIGURE 2.17 Stress concentration factors for uniform tensile bar with hole stress raiser

Courtesy Esposito - Machine Design, 1991, Delmar Publishers

Sample problem 2.1: Fig. 2.18 shows a rectangular bar containing a hole and undergoing a tensile force of 10,000 pounds. In this case t = 2 in., d = 0.5 in., and w = 1.5 in.
a) Find the tensile stress at no hole condition;
b) Find the average stress at the hole section assuming no stress concentration; and
c) Find the maximum stress at the hole section taking into consideration the effect of the stress raiser.

Stress concentration problem (Fig. 2.18)



Courtesy Esposito - Machine Design, 1991, Delmar Publishers

Solution

- a) Stress with no hole = 10000/ (2 × 1.5)
  = 333 psi
- b) $S_{avg} = \frac{F}{A_B - A_H}$

  $S_{avg} = \frac{10000}{2\times1.5 - 2 \times 0.5}$ = 5000 psi
- c) From Fig 2.17, K = 2.3 $S_{max} = S_{avg} \times K$

  $S_{max} = 5000 \times 2.3$ = 11,500 psi

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Reducing stress concentrations

- Make all transitions as gradual as possible. (Fig. 2.20)
- Drill holes on both sides of key way (Fig. 2.19)
- Thread root diameter equals the adjacent shank diameter. (Fig. 2.21)
- Add shoulders for press fitted components. (Fig. 2.22)

THOMSON DELMAR LEARNING Copyright © 2004 Thomson Delmar Learning

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Stress concentration Fig. 2.19)

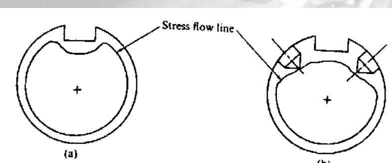


FIGURE 2.19 Stress reduction by drilling holes on both sides of keyway in shaft

Courtesy Esposito - Machine Design, 1991, Delmar Publishers

THOMSON DELMAR LEARNING Copyright © 2004 Thomson Delmar Learning

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Stress concentration Fig. 2.20)

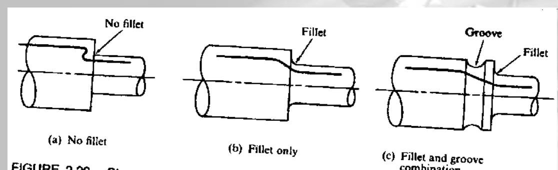


FIGURE 2.20 Stress reduction by use of fillet and groove

Courtesy Esposito - Machine Design, 1991, Delmar Publishers

THOMSON DELMAR LEARNING Copyright © 2004 Thomson Delmar Learning

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Stress concentration (Fig. 2.21)

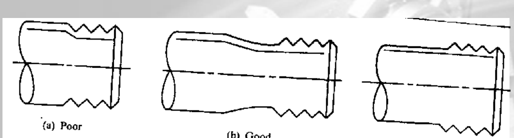


FIGURE 2.21 Stress concentration reduction for threaded cylindrical part

Courtesy Esposito - Machine Design, 1991, Delmar Publishers

THOMSON DELMAR LEARNING Copyright © 2004 Thomson Delmar Learning

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Stress concentration (Fig. 2.22)

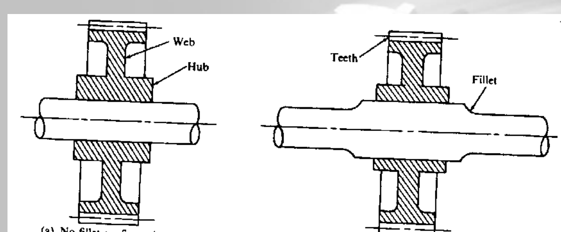


FIGURE 2.22 Reduction of stress concentration in gear press-fitted to shaft assembly

Courtesy Esposito - Machine Design, 1991, Delmar Publishers

THOMSON DELMAR LEARNING Copyright © 2004 Thomson Delmar Learning

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Allowable stress & Factor of Safety

$$S_{allowable} = \frac{S_{yield\_strength}}{FS_{ductile\_material}}$$

$$S_{allowable} = \frac{S_{ultimate\_strength}}{FS_{brittle\_material}}$$

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Uncertainity

- Variability of material composition and properties
- Effect of processing on properties
- Effect of nearby assemblies such as weldments
- Effect of thermo-mechanical treatment on properties

THOMSON DELMAR LEARNING Copyright © 2004 Thomson Delmar Learning

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Uncertainity

- Intensity and distribution of loading
- Validity of mathematical models used
- Intensity of stress concentrations
- Influence of time on strength and geometry
- Effect of corrosion
- Effect of wear

THOMSON DELMAR LEARNING Copyright © 2004 Thomson Delmar Learning

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Unknowns

- Exact type and magnitude of all loads
- Material property variations
- Precise stress concentration effects
- Extremes of environmental conditions, such as heat and moisture
- Approximate stress analysis formulae
- Residual stresses produced during manufacturing

THOMSON DELMAR LEARNING Copyright © 2004 Thomson Delmar Learning

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Factor of safety

- Determining a realistic factor of safety is very difficult.
- Higher factor increases cost and lower factor is likely to cause a premature failure.
- Hence

$$FS = a \times b \times c \times d$$

THOMSON DELMAR LEARNING Copyright © 2004 Thomson Delmar Learning

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Factor a

- Considers the variability of applied loads.

a = 1 for loads that are constant in magnitude and directions

a = 2 for complete load reversals in order to take fatigue into account

THOMSON DELMAR LEARNING Copyright © 2004 Thomson Delmar Learning

Factor b

- Considers the abruptness with which loads are applied.

b = 1 for gradually applied loads

b = 2 for suddenly applied loads

b = 3 or more (depending on the severity) for impact loads

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Factor c

- Considers the consequences of failure related to human safety, cost and so on.

c normally varies between 1.2 and 2 depending upon the seriousness of the failure

Copyright © 2004 Thomson Delmar Learning

MACHINE DESIGN TIMOTHY H. WENTZELL, P.E.

Factor d

- Differentiates FS ductile material from brittle material.

$$d = \frac{\text{Ultimate strength}}{\text{Yield strength}}$$

Copyright © 2004 Thomson Delmar Learning