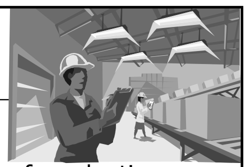
Inventory Control

Operations Management

Chapter 12

Chris Schrage OPS 12

Functions of Inventory



- Separate various aspects of production process *(decouple)*
- Protect against fluctuations in demand while providing a good selection of stock for customers
- Utilize quantity discounts to firm's advantage
- Hedge against inflation

Types of Inventory

- Raw materials
- Work-in-process (WIP)
- MRO
- Finished Goods inventory

Inventory Management

ABC Analysis
Record Accuracy
Cycle Counting
Control of Service Inventories

ABC Analysis

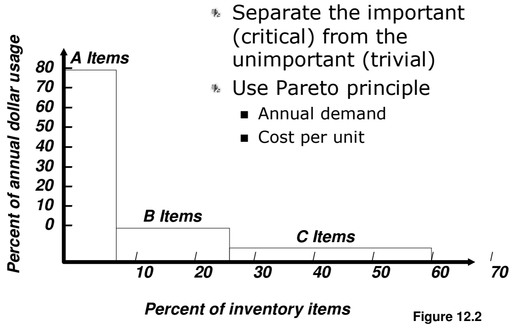
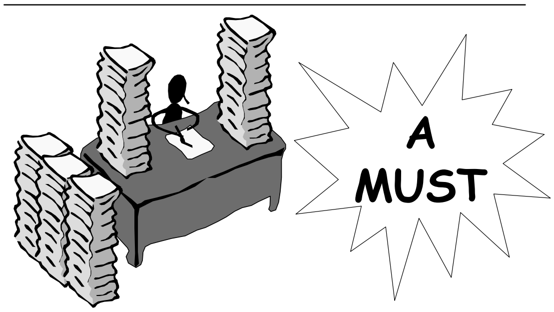


Figure 12.2

- Separate the important (critical) from the unimportant (trivial)
- Use Pareto principle
  - Annual demand
  - Cost per unit

Record Accuracy



Cycle Counting

- Continuous reconciliation of inventory with records
- In the past, factories would shut down for 1 to 2 days for a hands-on inventory count once a set time period
- Verification of records (computer age) streamlines process
- Advantages
  - Elimination of physical inventory counts disrupting production
  - Elimination of inventory adjustments
  - Accuracy audited by trained personnel
  - Errors identified and corrected in more timely manner
  - Accurate inventory records maintained

Service Inventories

- Retail
  - Restaurant
  - Consumer goods
- Shrinkage
- Pilferage
  - Shop-lifting
  - Usage of staff
- Control techniques
  - Personnel selection
  - Control of incoming shipments
  - Control of all goods leaving facility

Inventory Models- Costs

- Holding cost
  - Expense of keeping inventory in stock
- Ordering cost
  - Expense incurred during the ordering process
- Setup cost
  - Cost for retooling or resetting a machine to perform alternate tasks
- Setup time
  - Time spent retooling or resetting a machine to perform alternate tasks

Inventory Models

Independent Demand

Inventory Holding Costs
When to order
How much to order

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EOQ Economic Order quantity

- ***Demand is known, constant, and independent***
- ***Lead time is known and constant***
- ***Receipt of inventory is instantaneous and complete***
- ***Quantity discounts are not possible***
- ***Only variable costs are setup and holding***
- ***Stock-outs can be completely avoided***

Inventory Usage Over Time

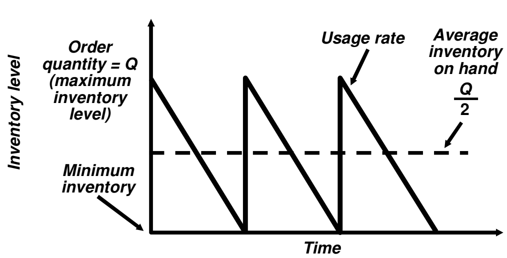


Figure 12.3

Minimizing Costs

Objective is to minimize total costs

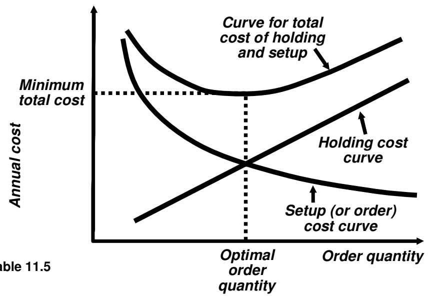


Table 11.5

The EOQ Model

Annual setup cost = $\frac{D}{Q}S$

Q = Number of pieces per order
Q^* = Optimal number of pieces per order (EOQ)
D = Annual demand in units for the Inventory item
S = Setup or ordering cost for each order
H = Holding or carrying cost per unit per year

Annual setup cost = (*Number of orders placed per year*) x (*Setup or order cost per order*)

$$= \left(\frac{\text{Annual demand}}{\text{Number of units in each order}}\right)\left(\begin{matrix}\text{Setup or order}\\ \text{cost per order}\end{matrix}\right)$$

$$= \left(\frac{D}{Q}\right)(S)$$

The EOQ Model

Annual setup cost = $\frac{D}{Q}S$

Annual holding cost = $\frac{Q}{2}H$

Q = Number of pieces per order
Q^* = Optimal number of pieces per order (EOQ)
D = Annual demand in units for the Inventory item
S = Setup or ordering cost for each order
H = Holding or carrying cost per unit per year

Annual holding cost = (*Average inventory level*) x (*Holding cost per unit per year*)

$$= \left(\frac{\text{Order quantity}}{2}\right)(\text{Holding cost per unit per year})$$

$$= \left(\frac{Q}{2}\right)(H)$$

The EOQ Model

Annual setup cost = $\frac{D}{Q}S$

Annual holding cost = $\frac{Q}{2}H$

Q = Number of pieces per order
*Q** = Optimal number of pieces per order (EOQ)
D = Annual demand in units for the Inventory item
S = Setup or ordering cost for each order
H = Holding or carrying cost per unit per year

Optimal order quantity is found when annual setup cost equals annual holding cost

$$\frac{D}{Q}S = \frac{Q}{2}H$$

*Solving for Q**

$$2DS = Q^2H$$
$$Q^2 = 2DS/H$$
$$Q^* = \sqrt{2DS/H}$$

An EOQ Example

Determine optimal number of needles to order
D = 1,000 *units*
S = $10 *per order*
H = $.50 *per unit per year*

$$Q^* = \sqrt{\frac{2DS}{H}}$$

$$Q^* = \sqrt{\frac{2(1{,}000)(10)}{0.50}} = \sqrt{40{,}000} = 200 \text{ units}$$

An EOQ Example

Determine optimal number of needles to order
D = 1,000 *units*   *Q** = 200 *units*
S = $10 *per order*
H = $.50 *per unit per year*

$$\text{Expected number of orders} = N = \frac{\text{Demand}}{\text{Order quantity}} = \frac{D}{Q^*}$$

$$N = \frac{1{,}000}{200} = 5 \text{ orders per year}$$

An EOQ Example

Determine optimal number of needles to order
D = 1,000 *units* *Q** = 200 *units*
S = $10 *per order* *N* = 5 *orders per year*
H = $.50 *per unit per year*

$$\text{Expected time between orders} = T = \frac{\text{Number of working days per year}}{N}$$

$$T = \frac{250}{5} = 50 \text{ days between orders}$$

An EOQ Example

Determine optimal number of needles to order
D = 1,000 *units* *Q** = 200 *units*
S = $10 *per order* *N* = 5 *orders per year*
H = $.50 *per unit per year* *T* = 50 *days*

Total annual cost = Setup cost + Holding cost

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$TC = \frac{1{,}000}{200}(\$10) + \frac{200}{2}(\$.50)$$

$$TC = (5)(\$10) + (100)(\$.50) = \$50 + \$50 = \$100$$

Robust Model

The EOQ model is robust

It works even if all parameters and assumptions are not met

The total cost curve is relatively flat in the area of the EOQ

Reorder Points

EOQ answers the "how much" question

The reorder point (ROP) tells when to order

$$ROP = \left(\textit{Demand per day}\right)\left(\textit{Lead time for a new order in days}\right)$$

$$= d \times L$$

$$d = \frac{D}{\text{Number of working days in a year}}$$

Reorder Point Curve

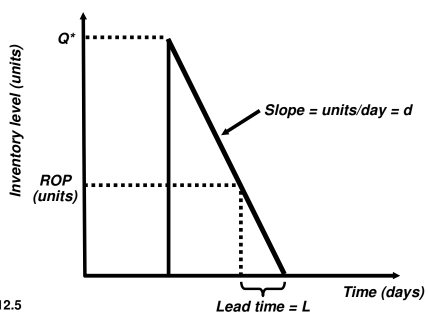


Figure 12.5

Reorder Point Example

Demand = 8,000 *DVDs per year*
250 *working day year*
Lead time for orders is 3 *working days*

$$d = \frac{D}{\text{Number of working days in a year}}$$

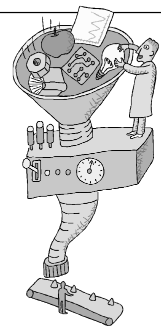
= 8,000/250 = 32 *units*

ROP = *d x L*

= 32 *units per day x* 3 *days* = 96 *units*

Production Order Quantity Model

- ***Used when inventory builds up over a period of time after an order is placed***
- ***Used when units are produced and sold simultaneously***



Production Order Quantity Model

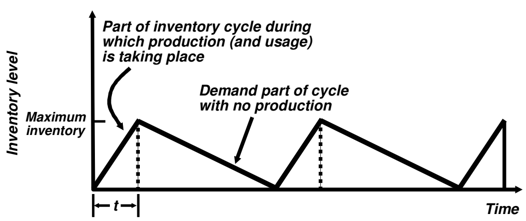


Figure 12.6

Production Order Quantity Model

Q = Number of pieces per order
H = Holding cost per unit per year
t = Length of the production run in days
p = Daily production rate
d = Daily demand/usage rate

$$\begin{pmatrix}\text{Annual inventory} \\ \text{holding cost}\end{pmatrix} = (\text{Average inventory level}) \times \begin{pmatrix}\text{Holding cost} \\ \text{per unit per year}\end{pmatrix}$$

$$\begin{pmatrix}\text{Annual inventory} \\ \text{level}\end{pmatrix} = (\text{Maximum inventory level})/2$$

$$\begin{pmatrix}\text{Maximum} \\ \text{inventory level}\end{pmatrix} = \begin{pmatrix}\text{Total produced during} \\ \text{the production run}\end{pmatrix} - \begin{pmatrix}\text{Total used during} \\ \text{the production run}\end{pmatrix}$$

$$= pt - dt$$

Production Order Quantity Model

Q = Number of pieces per order p = Daily production rate
H = Holding cost per unit per year d = Daily demand/usage rate
t = Length of the production run in days

$$\begin{pmatrix}\text{Maximum}\\ \text{inventory level}\end{pmatrix} = \begin{pmatrix}\text{Total produced during}\\ \text{the production run}\end{pmatrix} - \begin{pmatrix}\text{Total used during}\\ \text{the production run}\end{pmatrix}$$

$$= pt - dt$$

However, Q = total produced = pt ; thus $t = Q/p$

$$\begin{pmatrix}\text{Maximum}\\ \text{inventory level}\end{pmatrix} = p\left(\frac{Q}{p}\right) - d\left(\frac{Q}{p}\right) = Q\left(1 - \frac{d}{p}\right)$$

$$\text{Holding cost} = \frac{\text{Maximum inventory level}}{2}(H) = \frac{Q}{2}\left[1 - \left(\frac{d}{p}\right)\right]H$$

Production Order Quantity Model

Q = Number of pieces per order p = Daily production rate
H = Holding cost per unit per year d = Daily demand/usage rate
D = Annual demand

$$\text{Setup cost} = (D/Q)S$$
$$\text{Holding cost} = 1/2\ HQ[1 - (d/p)]$$

$$(D/Q)S = 1/2\ HQ[1 - (d/p)]$$

$$Q^2 = \frac{2DS}{H[1 - (d/p)]}$$

$$Q^* = \sqrt{\frac{2DS}{H[1 - (d/p)]}}$$

Production Order Quantity Example

D = 1,000 *units* p = 8 *units per day*
S = \$10 d = 4 *units per day*
H = \$0.50 *per unit per year*

$$Q^* = \sqrt{\frac{2DS}{H[1 - (d/p)]}}$$

$$Q^* = \sqrt{\frac{2(1{,}000)(10)}{0.50[1 - (4/8)]}} = \sqrt{80{,}000}$$

$$= 282.8 \text{ or } 283 \text{ hubcaps}$$

Production Order Quantity Model

When annual data are used the equation becomes

$$Q^* = \sqrt{\dfrac{2DS}{H\left(1 - \dfrac{\text{annual demand rate}}{\text{annual production rate}}\right)}}$$

Quantity Discount Models

Reduced prices are often available when larger quantities are purchased

Trade-off is between reduced product cost and increased holding cost

Total cost = Setup cost + Holding cost + Product cost

$$TC = \frac{D}{Q}S + \frac{QH}{2} + PD$$

Quantity Discount Models

A typical quantity discount schedule

Discount Number	Discount Quantity	Discount (%)	Discount Price (P)
1	*0* to *999*	no discount	*$5.00*
2	*1,000* to *1,999*	*4*	*$4.80*
3	*2,000* and over	*5*	*$4.75*

Table 12.2

Quantity Discount Models

Steps in analyzing a quantity discount

1. ***For each discount, calculate Q****
2. ***If Q* for a discount doesn't qualify, choose the smallest possible order size to get the discount***
3. ***Compute the total cost for each Q* or adjusted value from Step 2***
4. ***Select the Q* that gives the lowest total cost***

Quantity Discount Models

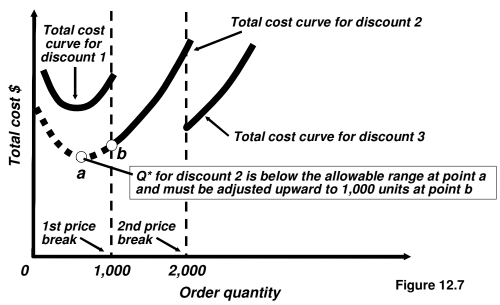


Figure 12.7

Quantity Discount Example

Calculate Q* for every discount

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

$$Q_1^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(5.00)}} = 700 \textit{ cars order}$$

$$Q_2^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(4.80)}} = 714 \textit{ cars order}$$

$$Q_3^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(4.75)}} = 718 \textit{ cars order}$$

Quantity Discount Example

Calculate Q for every discount*

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

$$Q_1^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(5.00)}} = 700 \text{ cars order}$$

$$Q_2^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(4.80)}} = \cancel{714} \text{ cars order}$$
1,000 — *adjusted*

$$Q_3^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(4.75)}} = \cancel{718} \text{ cars order}$$
2,000 — *adjusted*

Quantity Discount Example

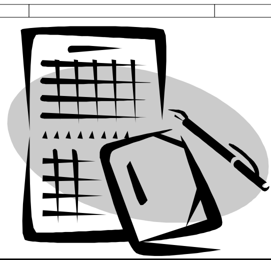
Discount Number	Unit Price	Order Quantity	Annual Product Cost	Annual Ordering Cost	Annual Holding Cost	Total
1	$5.00	700	$25,000	$350	$350	$25,700
2	$4.80	1,000	$24,000	$245	$480	$24,725
3	$4.75	2,000	$23.750	$122.50	$950	$24,822.50

Table 12.3

Choose the price and quantity that gives the lowest total cost

Buy 1,000 *units at* $4.80 *per unit*

Fixed Period Systems



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Assumptions

- Same as EOQ basic
  - Ordering and holding costs- only ones relevant
  - Lead times are known and constant
  - Items are independent
- Advantage
  - No physical counts of inventory after each withdrawal
- Disadvantage
  - Higher possibility of stock out
  - May need higher level of safety stock