Inventory Control

Operations Management
Chapter 12

Functions of Inventory

- Separate various aspects of production process (decouple)
- Protect against fluctuations in demand while providing a good selection of stock for customers
- Utilize quantity discounts to firm’s advantage
- Hedge against inflation

Types of Inventory

- Raw materials
- Work-in-process (WIP)
- MRO
- Finished Goods inventory
Inventory Management

ABC Analysis
Record Accuracy
Cycle Counting
Control of Service Inventories

ABC Analysis
- Separate the important (critical) from the unimportant (trivial)
- Use Pareto principle
  - Annual demand
  - Cost per unit

Record Accuracy

A MUST
Cycle Counting

- Continuous reconciliation of inventory with records
- In the past, factories would shut down for 1 to 2 days for a hands-on inventory count once a set time period
- Verification of records (computer age) streamlines process

Advantages
- Elimination of physical inventory counts disrupting production
- Elimination of inventory adjustments
- Accuracy audited by trained personnel
- Errors identified and corrected in more timely manner
- Accurate inventory records maintained

Service Inventories

- Retail
  - Restaurant
  - Consumer goods
- Shrinkage
- Pilferage
  - Shop-lifting
  - Usage of staff
- Control techniques
  - Personnel selection
  - Control of incoming shipments
  - Control of all goods leaving facility

Inventory Models - Costs

- Holding cost
  - Expense of keeping inventory in stock
- Ordering cost
  - Expense incurred during the ordering process
- Setup cost
  - Cost for retooling or resetting a machine to perform alternate tasks
- Setup time
  - Time spent retooling or resetting a machine to perform alternate tasks
Inventory Models

Independent Demand

Inventory Holding Costs
When to order
How much to order

EOQ  Economic Order quantity

- Demand is known, constant, and independent
- Lead time is known and constant
- Receipt of inventory is instantaneous and complete
- Quantity discounts are not possible
- Only variable costs are setup and holding
- Stock-outs can be completely avoided

Inventory Usage Over Time

Order quantity = \(Q\) (maximum inventory level)
Usage rate
Average inventory on hand \(\frac{Q}{2}\)
Minimum inventory

Figure 12.3
Minimizing Costs

Objective is to minimize total costs

![Graph showing minimum total cost and related costs]

Table 11.5

<table>
<thead>
<tr>
<th>Order quantity</th>
<th>Curve for total cost of holding and setup</th>
<th>Holding cost curve</th>
<th>Setup (or order) cost curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual cost</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The EOQ Model

Annual setup cost = \( \frac{D}{Q} \)

\( Q \) = Number of pieces per order
\( Q^* \) = Optimal number of pieces per order (EOQ)
\( D \) = Annual demand in units for the Inventory item
\( S \) = Setup or ordering cost for each order
\( H \) = Holding or carrying cost per unit per year

Annual setup cost = \( \left( \frac{\text{Number of orders placed per year}}{\text{Number of units in each order}} \right) \times \text{Setup or order cost per order} \)

\[ = \left( \frac{D}{Q} \right) \left( \frac{S}{\text{Number of units in each order}} \right) \]

Annual holding cost = \( \left( \frac{\text{Average inventory level}}{2} \right) \times \text{Holding cost per unit per year} \)

\[ = \left( \frac{Q}{2} \right) \left( \frac{D}{Q} \right) \left( \frac{S}{\text{Number of units in each order}} \right) \]

\[ = \frac{D}{2} \left( \frac{S}{\text{Number of units in each order}} \right) \]

\[ = \frac{D}{2} \left( \frac{S}{Q} \right) \]

\[ = \frac{D}{2} \times \frac{S}{Q} \]

\[ = \frac{D}{2} \times \frac{S}{Q} \]
The EOQ Model

\[ Q = \text{Number of pieces per order} \]
\[ Q^* = \text{Optimal number of pieces per order (EOQ)} \]
\[ D = \text{Annual demand in units for the inventory item} \]
\[ S = \text{Setup or ordering cost for each order} \]
\[ H = \text{Holding or carrying cost per unit per year} \]

Optimal order quantity is found when annual setup cost equals annual holding cost:

\[ \frac{D}{Q} S = \frac{Q^2 H}{2} \]

Solving for \( Q^* \):

\[ 2DS = Q^* H \]
\[ Q^* = \frac{2DS}{H} \]
\[ Q^* = \sqrt{\frac{2DS}{H}} \]

An EOQ Example

Determine optimal number of needles to order

\[ D = 1,000 \text{ units} \]
\[ S = $10 \text{ per order} \]
\[ H = $0.50 \text{ per unit per year} \]

\[ Q^* = \sqrt{\frac{2DS}{H}} \]
\[ Q^* = \sqrt{\frac{2(1,000)(10)}{0.50}} = \sqrt{40,000} = 200 \text{ units} \]

An EOQ Example

Determine optimal number of needles to order

\[ D = 1,000 \text{ units} \]
\[ Q^* = 200 \text{ units} \]
\[ S = $10 \text{ per order} \]
\[ H = $0.50 \text{ per unit per year} \]

\[ N = \frac{D}{Q^*} = \frac{1,000}{200} = 5 \text{ orders per year} \]
An EOQ Example

Determine optimal number of needles to order
\[ D = 1,000 \text{ units} \quad Q^* = 200 \text{ units} \]
\[ S = $10 \text{ per order} \quad N = 5 \text{ orders per year} \]
\[ H = $.50 \text{ per unit per year} \]

Expected time between orders = \( T = \frac{\text{Number of working days per year}}{N} \)

\[ T = \frac{250}{5} = 50 \text{ days between orders} \]

Total annual cost = Setup cost + Holding cost
\[ TC = S + \frac{Q}{2} H \]
\[ TC = (5)($10) + (200)($.50) = $50 + $50 = $100 \]

Robust Model

The EOQ model is robust
It works even if all parameters and assumptions are not met
The total cost curve is relatively flat in the area of the EOQ
Reorder Points

EOQ answers the “how much” question
The reorder point (ROP) tells when to order

\[ ROP = \left( \frac{\text{Demand per day}}{\text{Lead time for a new order in days}} \right) \]
\[ = d \times L \]
\[ d = \frac{D}{\text{Number of working days in a year}} \]

Reorder Point Curve

Figure 12.5

Reorder Point Example

Demand = 8,000 DVDs per year
250 working day year
Lead time for orders is 3 working days

\[ d = \frac{D}{\text{Number of working days in a year}} \]
\[ = \frac{8,000}{250} = 32 \text{ units} \]

\[ \text{ROP} = d \times L \]
\[ = 32 \text{ units per day} \times 3 \text{ days} = 96 \text{ units} \]
Production Order Quantity Model

* Used when inventory builds up over a period of time after an order is placed
* Used when units are produced and sold simultaneously

Figure 12.6

Production Order Quantity Model

\[ Q = \text{Number of pieces per order} \]
\[ p = \text{Daily production rate} \]
\[ H = \text{Holding cost per unit per year} \]
\[ d = \text{Daily demand/usage rate} \]
\[ t = \text{Length of the production run in days} \]

\[
\text{Annual inventory holding cost} = (\text{Average inventory level}) \times (\text{Holding cost per unit per year})
\]

\[
\text{Annual inventory level} = \frac{(\text{Maximum inventory level})}{2}
\]

\[
\text{Maximum inventory level} = pt - dt
\]

\[
\text{Total produced during the production run} - \text{Total used during the production run}
\]
Production Order Quantity Model

Q = Number of pieces per order  \( p = \text{Daily production rate} \)

H = Holding cost per unit per year  \( d = \text{Daily demand/usage rate} \)

\( t = \text{Length of the production run in days} \)

\( \text{Maximum inventory level} = \left( \frac{\text{Total produced during the production run}}{\text{Total used during the production run}} \right) = pt - dt \)

However,  \( Q = \text{total produced} = pt \); thus  \( t = Q/p \)

\( \text{Maximum inventory level} = p \left( \frac{Q}{p} \right) - d \left( \frac{Q}{p} \right) = Q - \frac{d}{p} \)

\[ \text{Maximum inventory level} = \frac{Q}{2} \left( 1 - \frac{d}{p} \right) \]

\[ \frac{1}{2}HQ[1 - \left( \frac{d}{p} \right)] \]

\( H \)

Production Order Quantity Model

Q = Number of pieces per order  \( p = \text{Daily production rate} \)

H = Holding cost per unit per year  \( d = \text{Daily demand/usage rate} \)

\( D = \text{Annual demand} \)

\( S = \text{Setup cost} \)

Setup cost = \( \left( \frac{D}{D} \right)Q \)

\( \text{Holding cost} = \frac{1}{2}HQ[1 - \left( \frac{d}{p} \right)] \)

\( Q^2 = \frac{2DS}{H[1 - \left( \frac{d}{p} \right)]} \)

\[ Q^* = \sqrt{\frac{2DS}{H[1 - \left( \frac{d}{p} \right)]}} \]

Production Order Quantity Example

\( D = 1,000 \text{ units} \)  \( p = 8 \text{ units per day} \)
\( S = \$10 \)  \( d = 4 \text{ units per day} \)
\( H = \$0.50 \text{ per unit per year} \)

\[ Q^* = \sqrt{\frac{2DS}{H[1 - \left( \frac{d}{p} \right)]}} \]

\[ Q^* = \sqrt{\frac{2(1,000)(10)}{0.50[1 - (4/8)]}} = \sqrt{80,000} \]

\[ Q^* = 282.8 \text{ or 283 hubcaps} \]
When annual data are used the equation becomes

\[ Q^* = \sqrt{\frac{2DS}{H\left(1 - \frac{\text{annual demand rate}}{\text{annual production rate}}\right)}} \]

Reduced prices are often available when larger quantities are purchased.

Trade-off is between reduced product cost and increased holding cost.

Total cost = Setup cost + Holding cost + Product cost

\[ TC = \frac{D}{Q} S + \frac{QH}{2} + PD \]

A typical quantity discount schedule

<table>
<thead>
<tr>
<th>Discount Number</th>
<th>Discount Quantity</th>
<th>Discount (%)</th>
<th>Discount Price (P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 to 999</td>
<td>no discount</td>
<td>$5.00</td>
</tr>
<tr>
<td>2</td>
<td>1,000 to 1,999</td>
<td>4</td>
<td>$4.80</td>
</tr>
<tr>
<td>3</td>
<td>2,000 and over</td>
<td>5</td>
<td>$4.75</td>
</tr>
</tbody>
</table>

Table 12.2
Quantity Discount Models

Steps in analyzing a quantity discount

1. For each discount, calculate $Q^*$
2. If $Q^*$ for a discount doesn’t qualify, choose the smallest possible order size to get the discount
3. Compute the total cost for each $Q^*$ or adjusted value from Step 2
4. Select the $Q^*$ that gives the lowest total cost

**Figure 12.7**

<table>
<thead>
<tr>
<th>Order quantity</th>
<th>Total cost curve for discount 1</th>
<th>Total cost curve for discount 2</th>
<th>Total cost curve for discount 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2,000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Q* for discount 2 is below the allowable range at point a and must be adjusted upward to 1,000 units at point b.

Quantity Discount Example

Calculate $Q^*$ for every discount

\[ Q^* = \sqrt{\frac{2DS}{IP}} \]

\[ Q_{1^*} = \sqrt{\frac{2(5,000)(49)}{(.2)(5.00)}} = 700 \text{ cars order} \]

\[ Q_{2^*} = \sqrt{\frac{2(5,000)(49)}{(.2)(4.80)}} = 714 \text{ cars order} \]

\[ Q_{3^*} = \sqrt{\frac{2(5,000)(49)}{(.2)(4.75)}} = 718 \text{ cars order} \]
Quantity Discount Example

Calculate $Q^*$ for every discount

$Q^* = \sqrt{\frac{2DS}{IP}}$

$Q_1^* = \sqrt{\frac{2(5,000)(49)}{(0.2)(5.00)}} = 700 \text{ cars order}$

$Q_2^* = \sqrt{\frac{2(5,000)(49)}{(0.2)(4.80)}} = 714 \text{ cars order (1,000 adjusted)}$

$Q_3^* = \sqrt{\frac{2(5,000)(49)}{(0.2)(4.75)}} = 718 \text{ cars order (2,000 adjusted)}$

Quantity Discount Example

<table>
<thead>
<tr>
<th>Discount Number</th>
<th>Unit Price</th>
<th>Order Quantity</th>
<th>Annual Product Cost</th>
<th>Annual Ordering Cost</th>
<th>Annual Holding Cost</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5.00</td>
<td>700</td>
<td>$25,000</td>
<td>$350</td>
<td>$350</td>
<td>$25,700</td>
</tr>
<tr>
<td>2</td>
<td>$4.80</td>
<td>1,000</td>
<td>$24,000</td>
<td>$245</td>
<td>$480</td>
<td>$24,725</td>
</tr>
<tr>
<td>3</td>
<td>$4.75</td>
<td>2,000</td>
<td>$23,750</td>
<td>$122.50</td>
<td>$950</td>
<td>$24,822.50</td>
</tr>
</tbody>
</table>

Choose the price and quantity that gives the lowest total cost

Buy 1,000 units at $4.80 per unit

Fixed Period Systems
Assumptions

- Same as EOQ basic
  - Ordering and holding costs - only ones relevant
  - Lead times are known and constant
  - Items are independent
- Advantage
  - No physical counts of inventory after each withdrawal
- Disadvantage
  - Higher possibility of stock out
  - May need higher level of safety stock